

# Two-Dimensional Superconductivity

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## Outline

1. Introduction
2. Various Superconductor-Insulator Transitions
2. Electrostatic Charging to Induce Superconductivity
3. Observations

## Introduction

Does 2D superconductivity exist and if it does will it occur at elevated temperatures? (V. L. Ginzburg and D. A. Kirzhnits, Zh. Exp. Teor. Fiz. 46, 397 (1964))

Proposals:

electrons in surface energy levels  
superconductivity at interfaces

The study thin films was originally thought to be a route to high  $T_c$

It did not turn out to be the case, but it has been interesting:

fluctuations: equilibrium long-range order is not possible in 2 dimensions. Instead have quasi-long range order: Kosterlitz-Thouless Berezinskii or topological phase transition.

importance of disorder: superconductor-insulator transitions.

## Films: Dirty Superconductors

Ultrathin Quench-Deposited Films of Metals-unit cell thickness

Shal'nikov (1940s), Buckel, Hilsch, Glover, (1950s and 60s)

Strongin, Dynes, AMG, Valles, Xiong, and Wu

Films of MoGe,  $\text{In}_2\text{O}_3$  and TiN

Beasley, Hebard, Ovadyahu, Kapitulnik, Gantmakher, Shahar,  
Baturina, and others.

## Disorder and Superconductivity

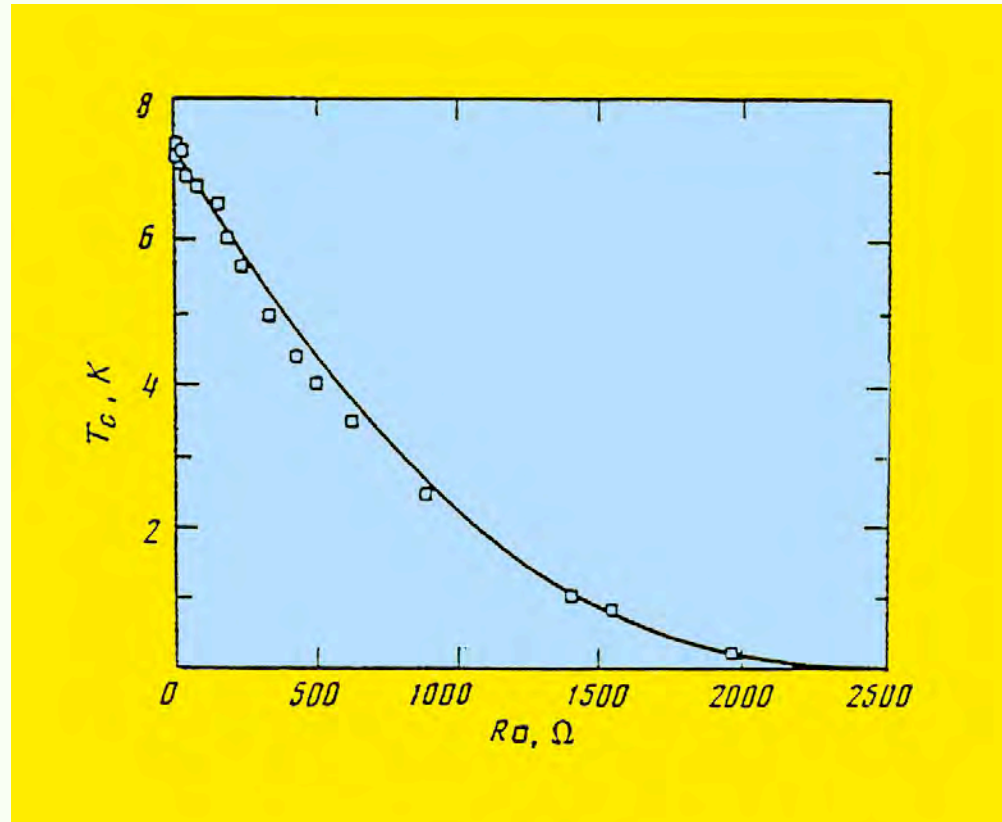
Early theories of dirty superconductors work in the low-disorder regime (**Anderson's Theorem**)

With a high enough level of disorder, **Anderson localization** occurs.

The effect of strong disorder on superconductivity involves both interactions and disorder.

Under strong conditions of electron localization, superconductivity should disappear, even with an attractive interaction.

## Transition temperature vs. sheet resistance in $\text{Mo}_x\text{Ge}_y$



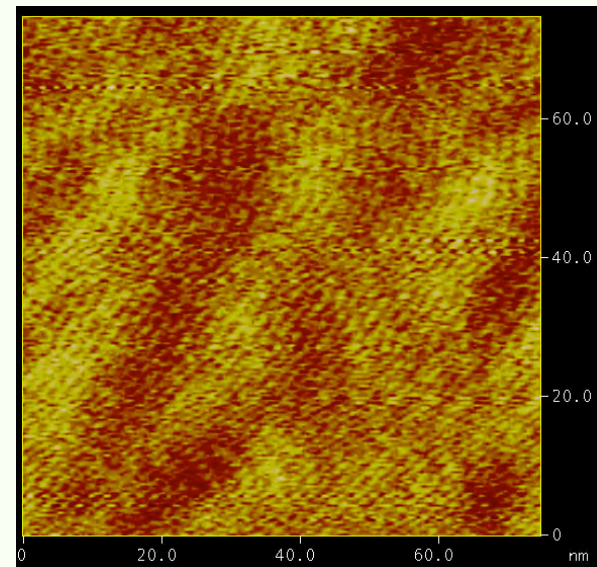
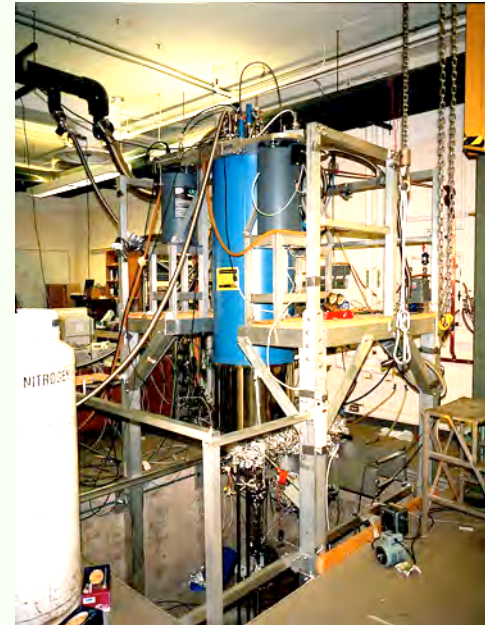
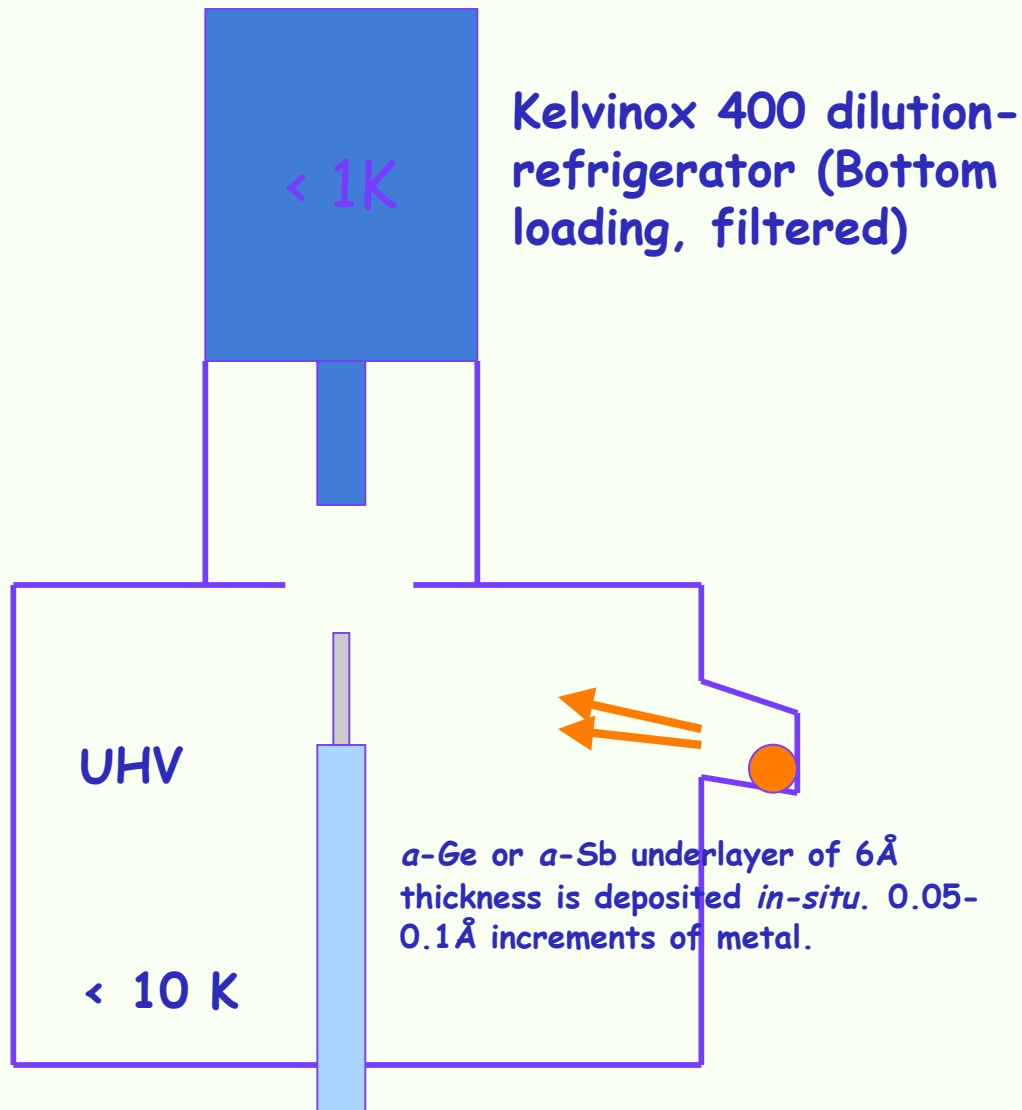
Experiment by Graybeal and Beasley (1984), theory by Finkel'shtein

## Does anything additional happen?

The model used to describe the data on the previous slide should not work in the limit of really strong disorder.

As the transition is driven down towards  $T = 0$ , with increasing disorder, fluctuations increase, and eventually they should be quantum mechanical.

# Apparatus for Quench-Condensation



AFM of film: RMS roughness of 0.3 Å, with maximal height deviations between dark and light of 3 Å

L.M. Hernandez and A.M. Goldman, Rev. Sci. Instrum. 73, 162 (2002)



## Films Grown on *a*-Ge Substrates - Homogeneous

Cyclic evaporation leads to evolution of superconductivity with thickness.

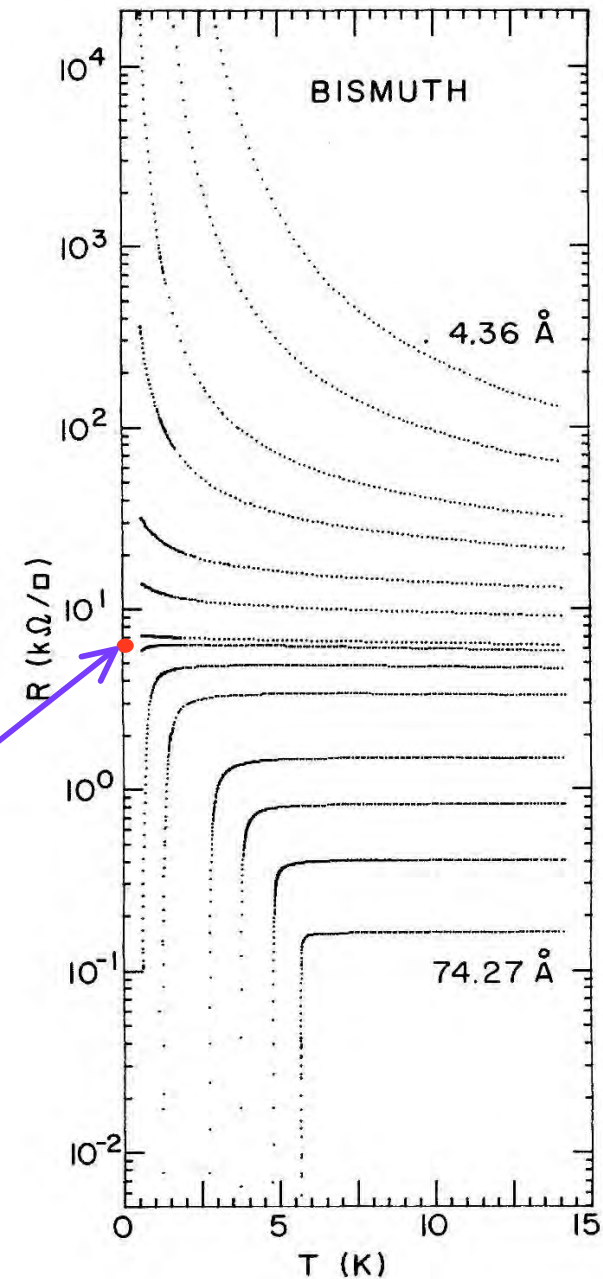
Apparent separation between superconducting and insulating behavior.

Critical resistance close to  $h/4e^2 = 6450 \Omega$

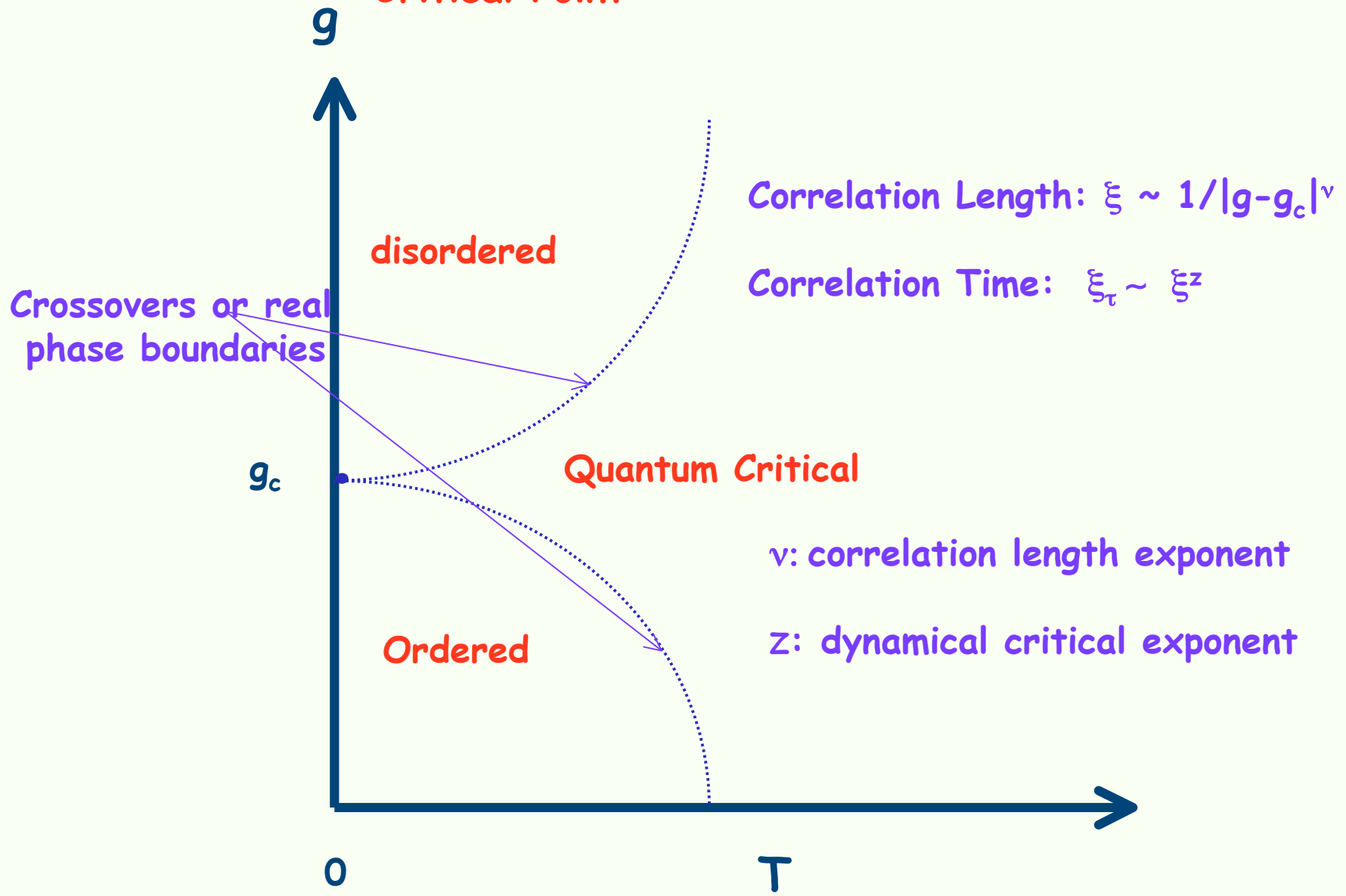
Curves of  $R(T)$  at different thicknesses look like renormalization flows.

Data Suggests: Quantum Critical Point (QCP) or zero-temperature quantum phase transition

Haviland, Liu, and Goldman  
Phys. Rev. Lett. 62, 2180 (1989)



# Possible Phase Diagram of a System Exhibiting a Quantum Critical Point



# Determination of QCP in 2D from Measurements at Nonzero Temperature

Have divergent correlation lengths, for the spatial,  $\xi$ , and for the temporal direction,  $\xi_\tau$ . The latter is associated with a vanishing energy scale

$$\xi \sim |\delta g|^{-\nu}, \text{ and } \xi_\tau \sim \xi^z$$

The correlation length exponent is  $\nu$  and the dynamical critical exponent is  $z$ .

After some analysis, the finite size scaling form for resistance in two dimensions is:

$$R_\square = R_c F(\delta g/T^{1/\nu z})$$

The control parameter is  $\delta g = |d-d_c|$ ,  $|H - H_c|$  or  $|n - n_c|$

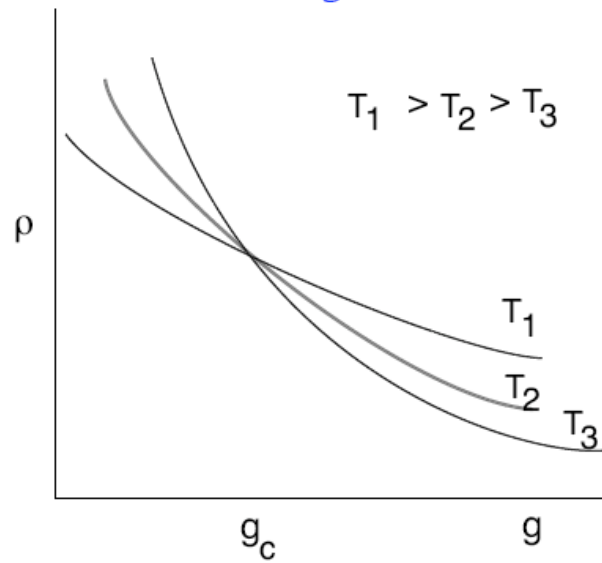
**Additional feature:** nonlinear response from quantum fluctuations

$$R_\square = R_c F(\delta g/T^{1/\nu z}, \delta g/E^{1/\nu(z+1)})$$

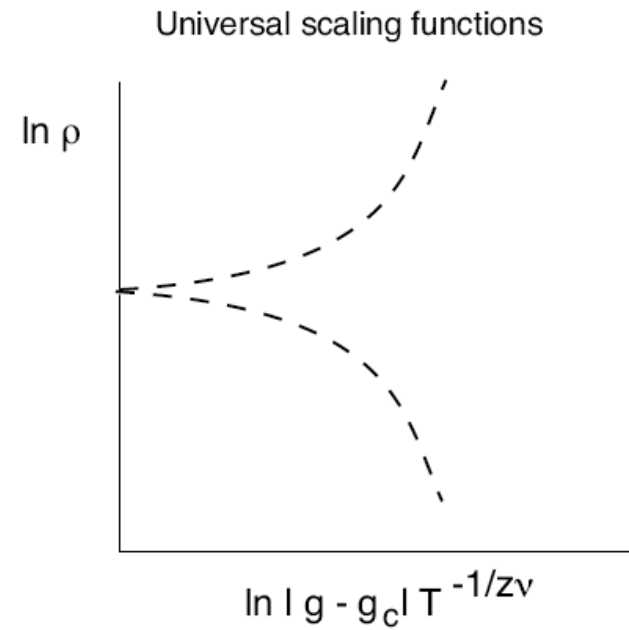
M. P. A. Fisher, PRL (1990)

# Analysis of Data

## Determination of $g_c$



## Data Collapse



## Quantum Phase Transition

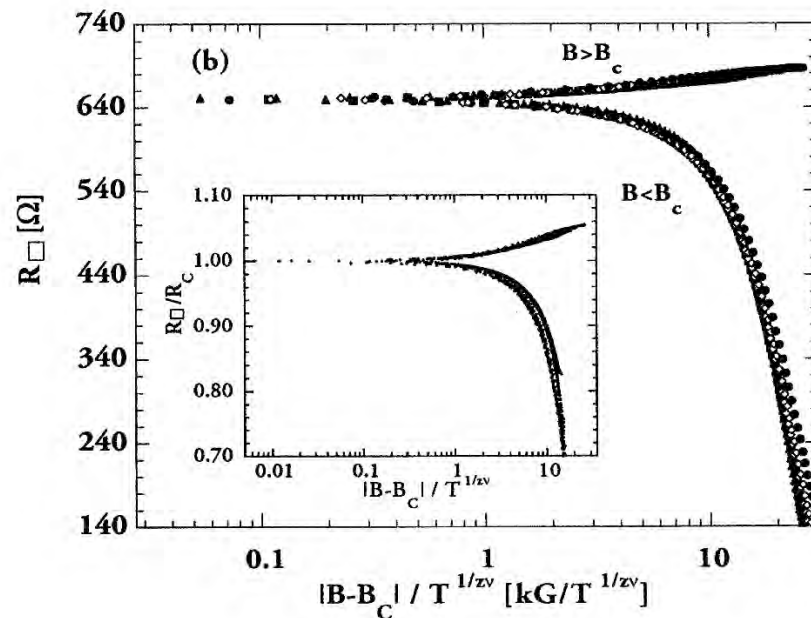
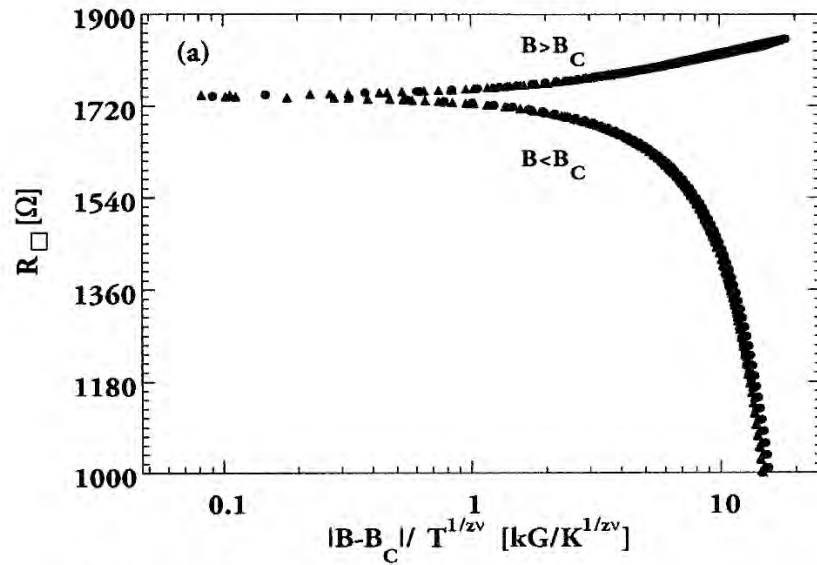
Scaling of the conductivity  
Critical Exponents  
Universality Class  
Universal Critical Resistance

## Nature of the Insulating Phase

Are the Cooper pairs in the insulating phase?  
Fermi or Bose Insulator?  
Is the amplitude of the superconducting order parameter zero?

## Role of disorder and dissipation at the transition?

# Field Driven Transition of $\text{Mo}_x\text{Ge}_y$



(Kapitulnik and Yazdani,  
First done by Hebard  
and Palaanen)

From scaling,  
 $zv = 4/3$   
and from electric  
field scaling  $z = 1$

# Scaling Tests - Amorphous Bi

## Thickness Scaling

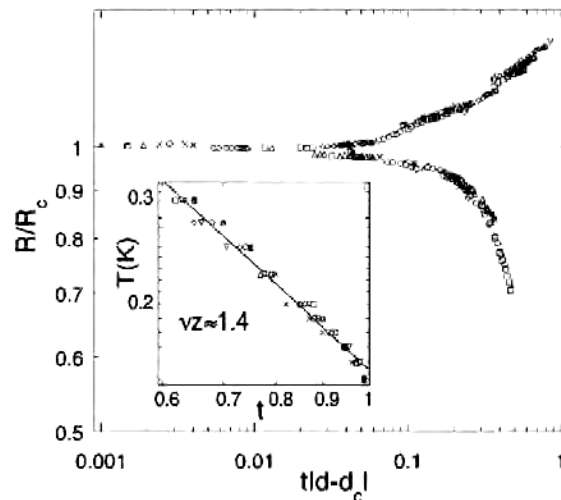


FIG. 7. Normalized resistance per square as a function of the scaling variable  $t|d-d_c|$  in different magnetic fields: 0.5 (squares) 1.0 (circles), 3.0 (crosses), 4.5 (triangles), and 7.0 kG (diamonds) Inset: The fitting of a power law to the temperature dependence of the parameter  $t$  determines the value of  $\nu z$ .

$$t = T^{-1/\nu z}$$

$\nu$  is the correlation length exponent  
 $z$  is the dynamical critical exponent

## Field Scaling

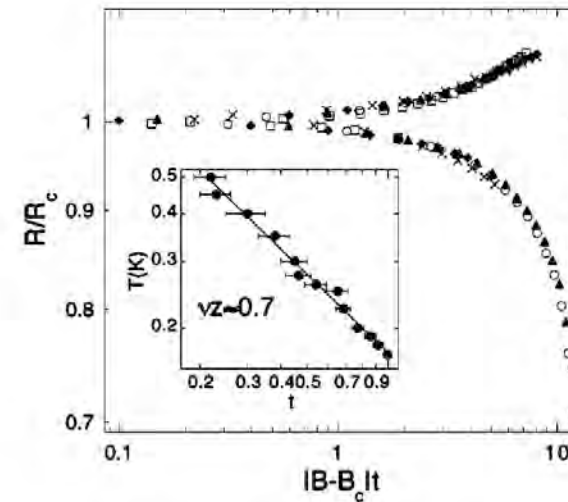


FIG. 3. Normalized resistance per square as a function of the scaling variable  $T^{-1/\nu z} |B - B_c|$ . Each symbol represents one film at different temperatures (only a small portion of the data is shown for clarity). Inset: The fitting a power law to the temperature dependence of the parameter  $t$  determines the value of  $\nu z$ .

(N. Markovic *et al.*, PRB (1999))

# The Critical Resistance is not Universal

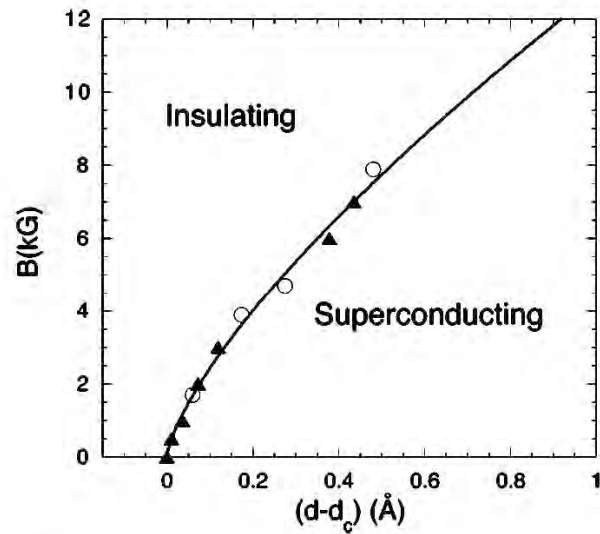


FIG. 9. The phase diagram in the  $d$ - $B$  plane in the  $T=0$  limit. The points on the phase boundary were obtained from thickness tuned transitions (triangles) and magnetic-field-tuned transitions (circles). The solid line is a power-law fit. Here  $d_c$  is taken to be the critical thickness in zero field.

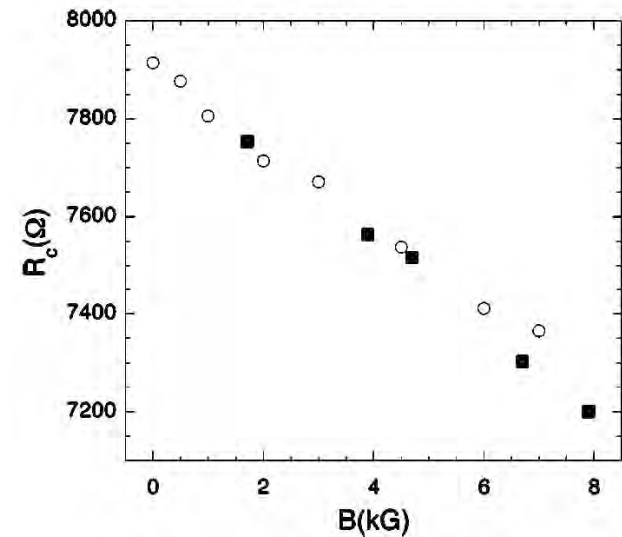


FIG. 10. The critical resistance as a function of the critical field for a series of bismuth films. Here  $R_c$  decreases with increasing thickness, as thicker films have lower normal-state resistances and higher critical fields.



# Field-Tuned SI Transition of InO<sub>x</sub> Films (Hebard and Paalanen) Bose Insulator, or ??

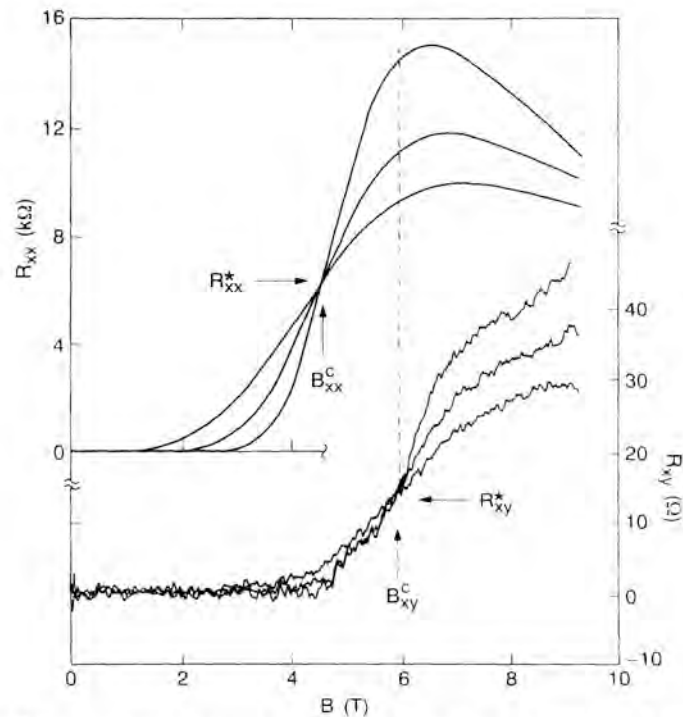


FIG. 4. Field dependence of  $R_{xx}$  (left-hand axis) and  $R_{xy}$  (right-hand axis) for the same film showing the separation between  $B_{xx}^c$  and  $B_{xy}^c$ .

Paalanen, Hebard, and Ruel  
Phys. Rev. Lett. 69, 1604 (1992)

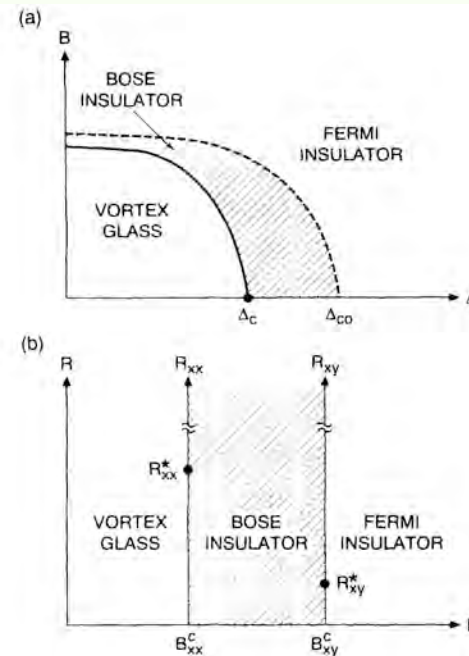
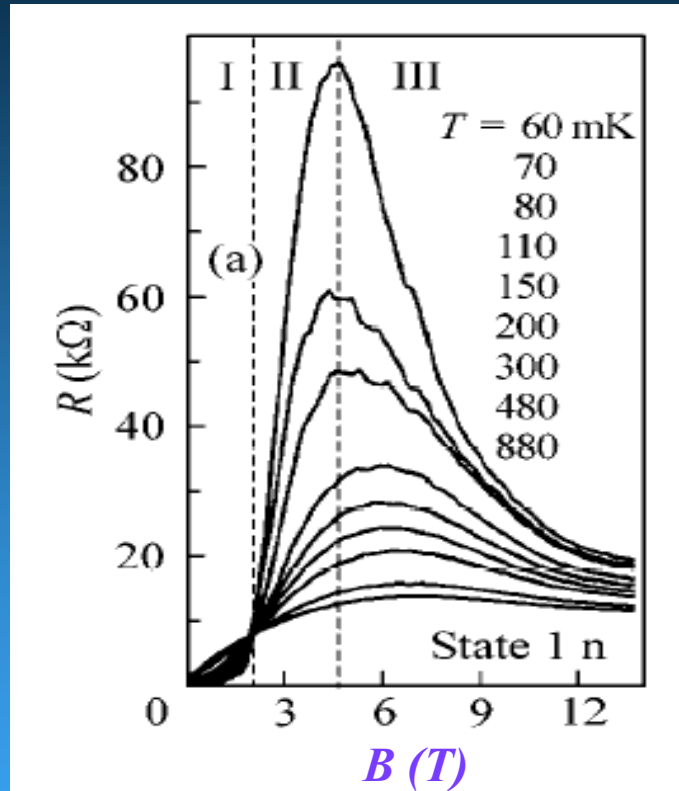
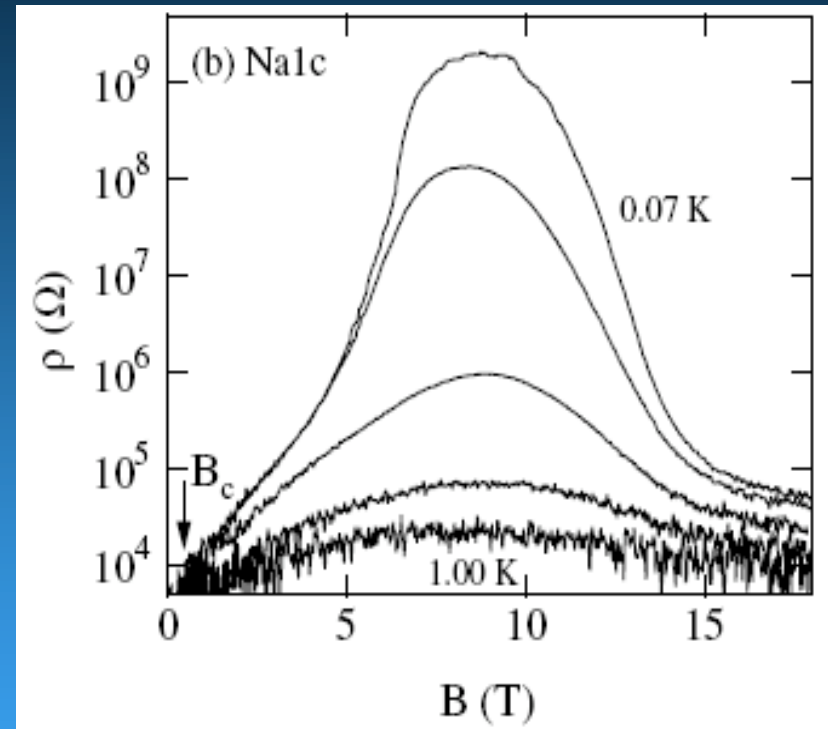


FIG. 1. Schematic showing the relationship between the vortex-glass, Bose-insulator, and Fermi-insulator phases of a 2D superconductor in a perpendicular magnetic field  $B$ . (a) The separate phases as a function of field  $B$  and disorder  $\Delta$ , with critical disorder  $\Delta_c$  marking the  $B=0$  superconducting-insulating transition and  $\Delta_{c0}$  marking the  $B=0$  disappearance of localized pairs. (b) The divergences of  $R_{xx}$  and  $R_{xy}$  through critical resistances  $R_{xx}^*$  and  $R_{xy}^*$  and at critical fields  $B_{xx}^c$  and  $B_{xy}^c$ , respectively.

# Magnetic Field-Tuned SIT in $\alpha$ - $\text{In}_x\text{O}_y$ Films



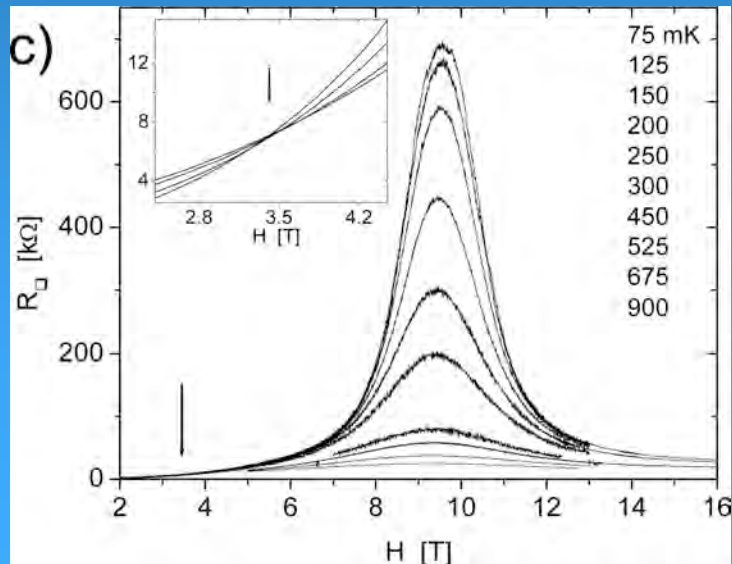
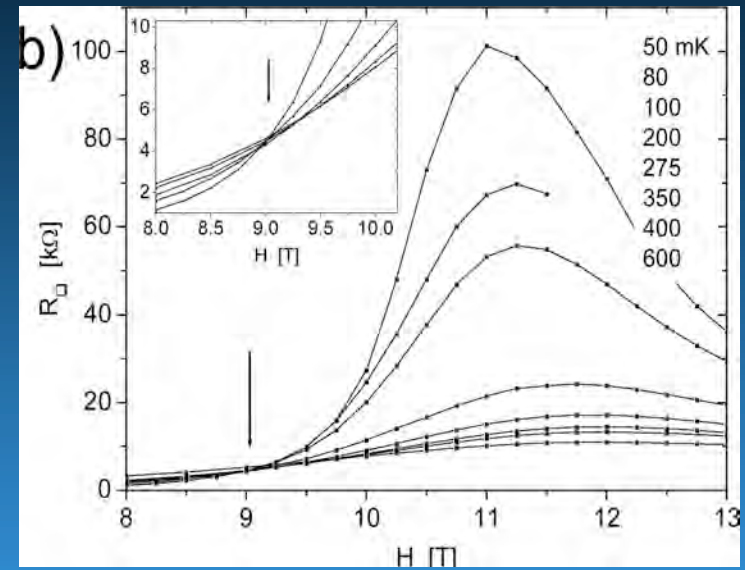
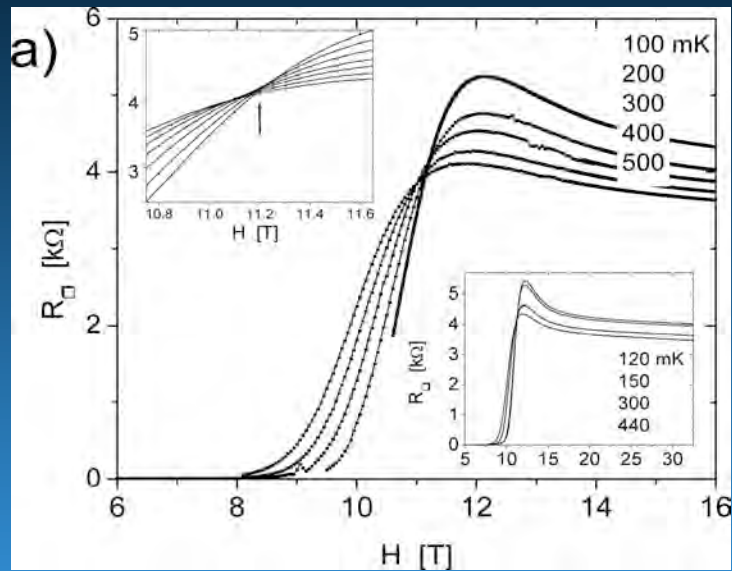
[Gantmakher, *et al.*, 2000]



[Sambandamurthy, *et al.*, 2004]

**Material, microstructure, disorder, carrier concentration determine the magnitude of the resistance peak**

## Other Studies of SIT in $\alpha$ - $\text{In}_x\text{O}_y$ Films



a: Weak insulator ( $R_N = 2.6 \text{ k}\Omega$ )

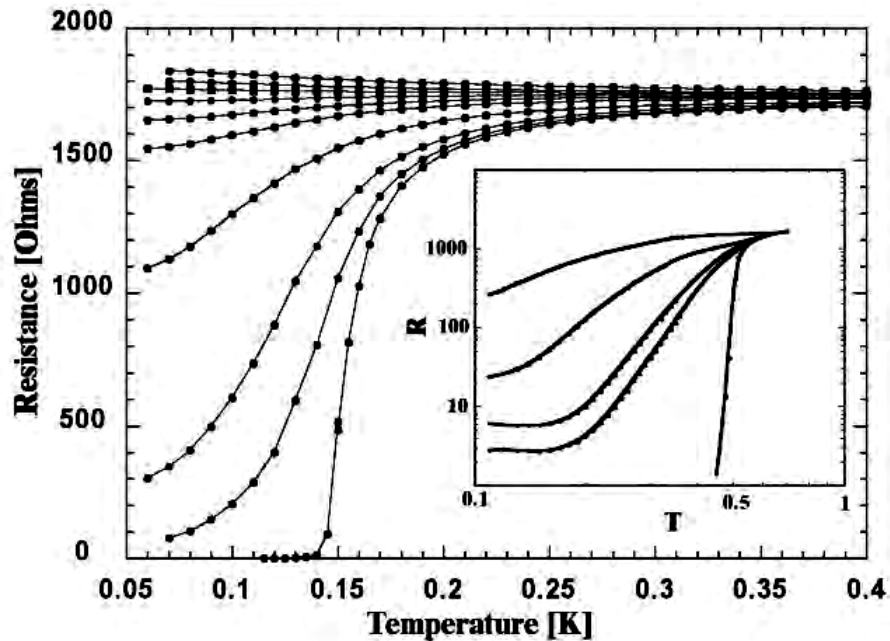
b: Intermediate insulator ( $R_N = 5 \text{ k}\Omega$ )

c: Strong insulator ( $R_N = 6.8 \text{ k}\Omega$ )

[Steiner and Kapitulnik, 2005]

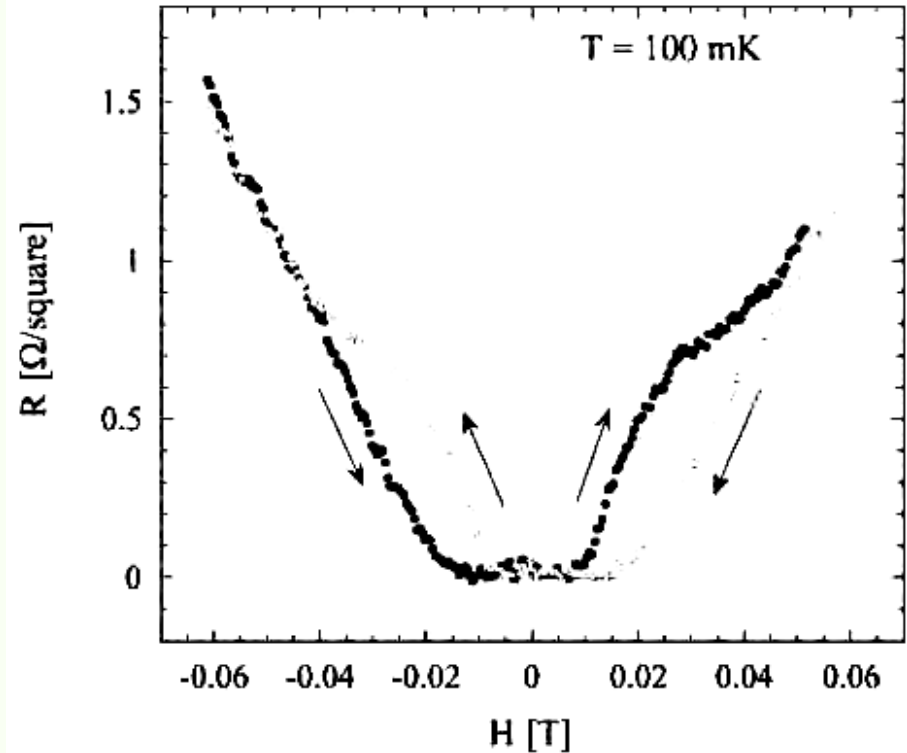
# Metallic Behavior in Field for $\text{Mo}_x\text{Ge}_y$ - breakdown of scaling at low T

Mason and Kapitulnik



Main curve:  $B = 0, 0.5, 1.0, 2, 3, 4, 4.4, 4.5, 5.5,$  and  $6$  kOe

Inset:  $B = 0, 1.5, 2.4,$  and  $7$  kOe for a second sample. Data suggest that the low temperature limit is metallic rather than superconducting.



Magnetoresistance at  $0.08\text{K}$  of a  $\text{Mo}_{43}\text{Ge}_{57}$  sandwich between layers of  $\alpha\text{-Ge}$

Scaling works at high temperatures, fails at low temperature.

# SIT in 2D Metal Films

- **SIT driven by amplitude fluctuations of the superconducting order parameter.**  
Maekawa and Fukuyama 1981, Finkelshtein 1987, Belitz 1989  
Cooper pairing suppressed by disorder and enhanced  $e$ - $e$  interactions  
Superconducting gap vanishes in the insulating phase, fermionic insulator  
Trivedi *et al.* 1990, Huscroft *et al.* 1998 and Ghosal *et al.* 2002  
Monte Carlo simulation of attractive Hubbard model with on-site interaction. Inhomogeneous pairing leads to a finite gap even in the insulating phase. Also: Dubi, Avishai, and Meir, 2007.
- **SIT driven by phase fluctuations of the order parameter**  
Fisher *et al.* 1989, 1990, Interacting boson model with disorder.  
Superconducting phase has itinerant Cooper pairs and localized vortices.  
Insulating phase has itinerant vortices and localized Cooper pairs  
SIT has a universal value of  $R_c$  with specific values of the critical exponents
- **Classical percolation model of the SIT**  
Shimshoni *et al.* 1998, Mason *et al.* 1999  
Film contains superconducting and insulating puddles resulting in an intermediate metallic regime and a saturation of resistance at low temperatures.  $\nu z = 4/3$

**Dissipation controlled transition ?** Chakravarty and Kivelson and others

## Role of Disorder

(Ghosal, Randeria and Trivedi, PRB 65, 014501 (2002))  
Inhomogeneous pairing in disordered *s*-wave superconductors

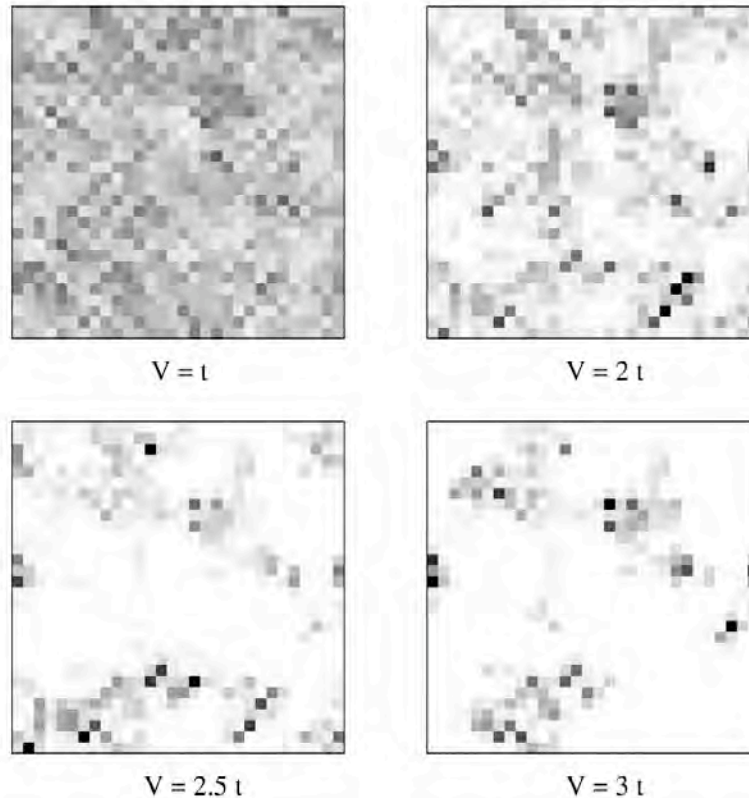
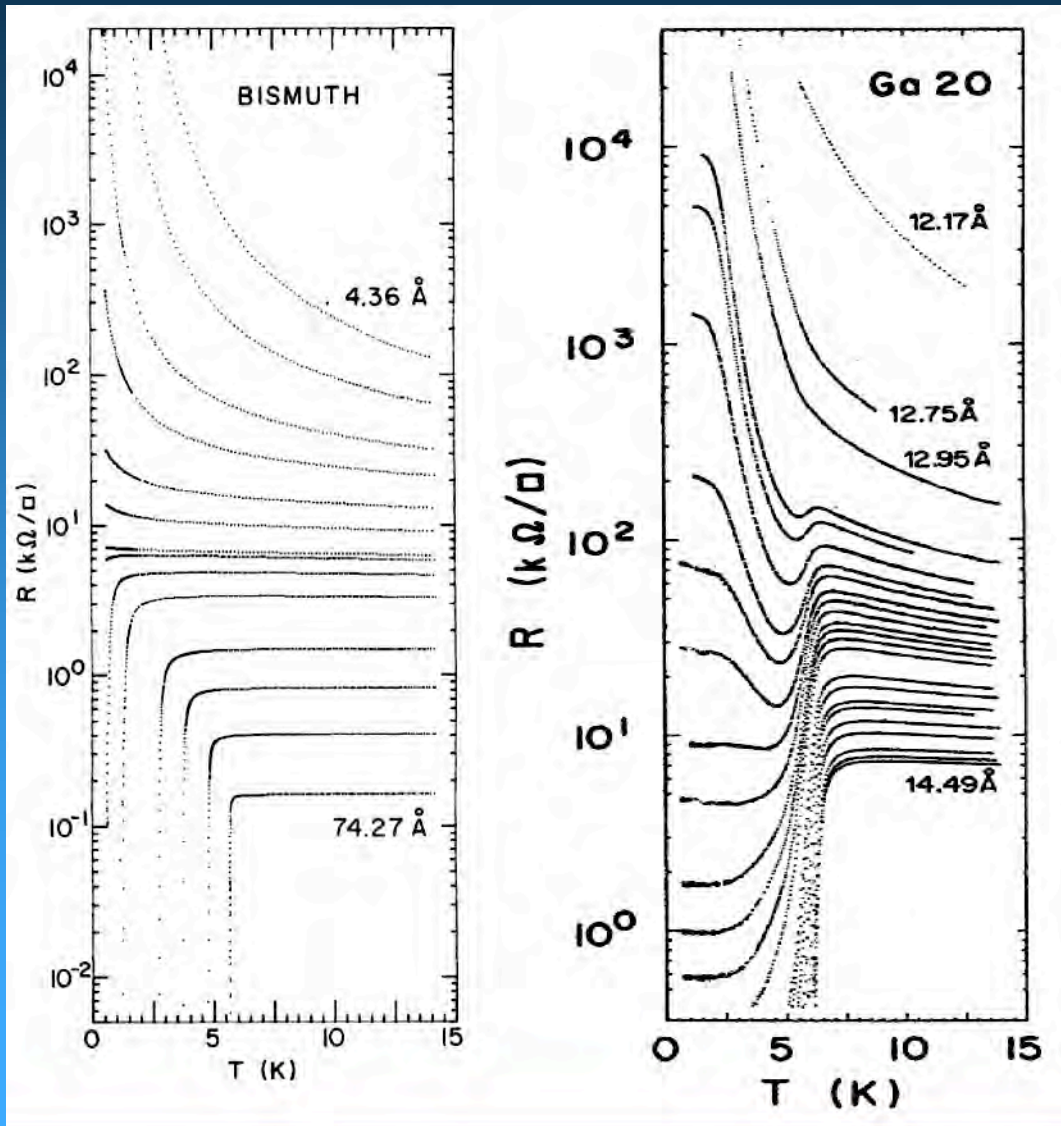


FIG. 6. Gray-scale plot for the spatial variation of the local pairing amplitude  $\Delta(\mathbf{r})$  for a particular realization of the random potential (same in all the panels) but with increasing disorder strength. Note that at large  $V$  the system generates “SC islands” (dark regions) with large pairing amplitude separated by an insulating “sea” (white regions) with negligible pairing amplitude.

There is some evidence that this happens in films.

# Homogeneous Amorphous vs. Granular



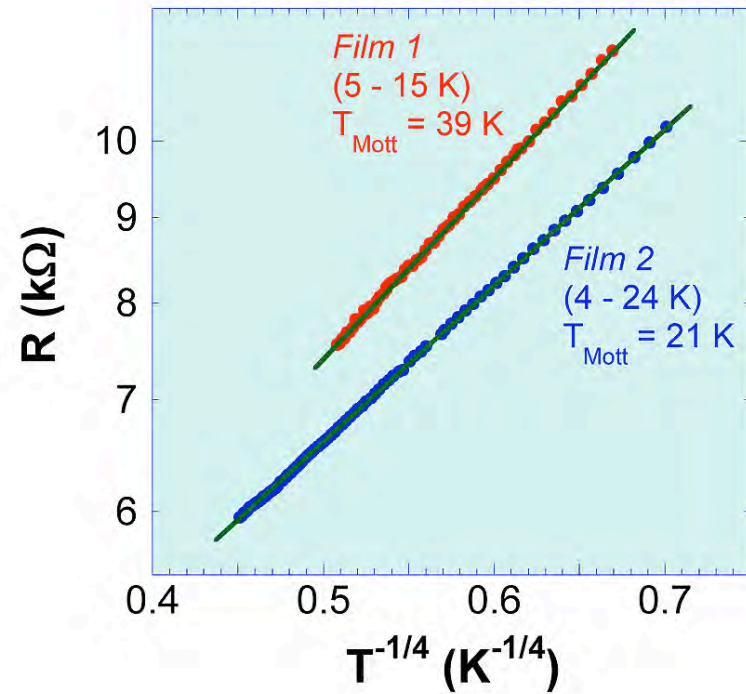
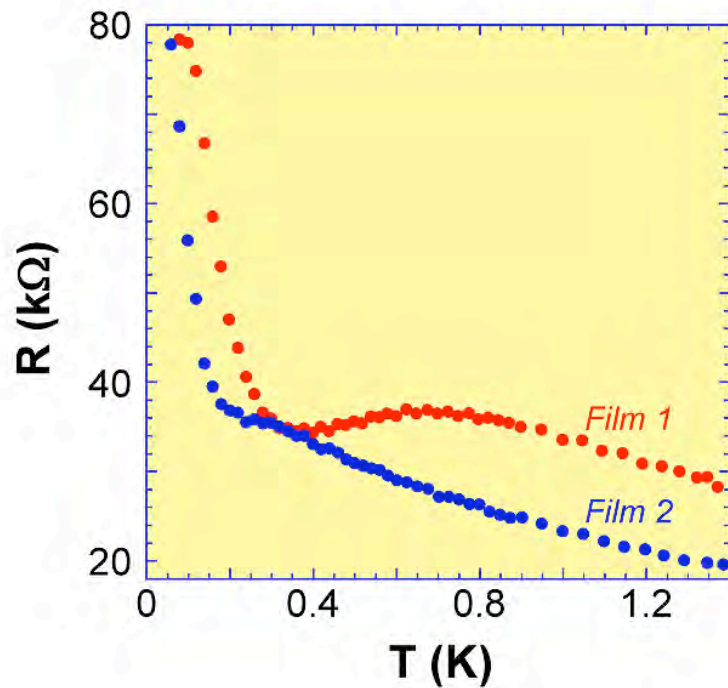
**Left graph:**

Bi film grown onto  
amorphous Ge  
underlayer on a glazed  
 $\text{Al}_2\text{O}_3$  substrate  
[Haviland, *et al.*, 1989]

**Right graph:**

Ga film deposited  
directly onto a glazed  
 $\text{Al}_2\text{O}_3$  substrate  
[Jaeger, *et al.*, 1989]

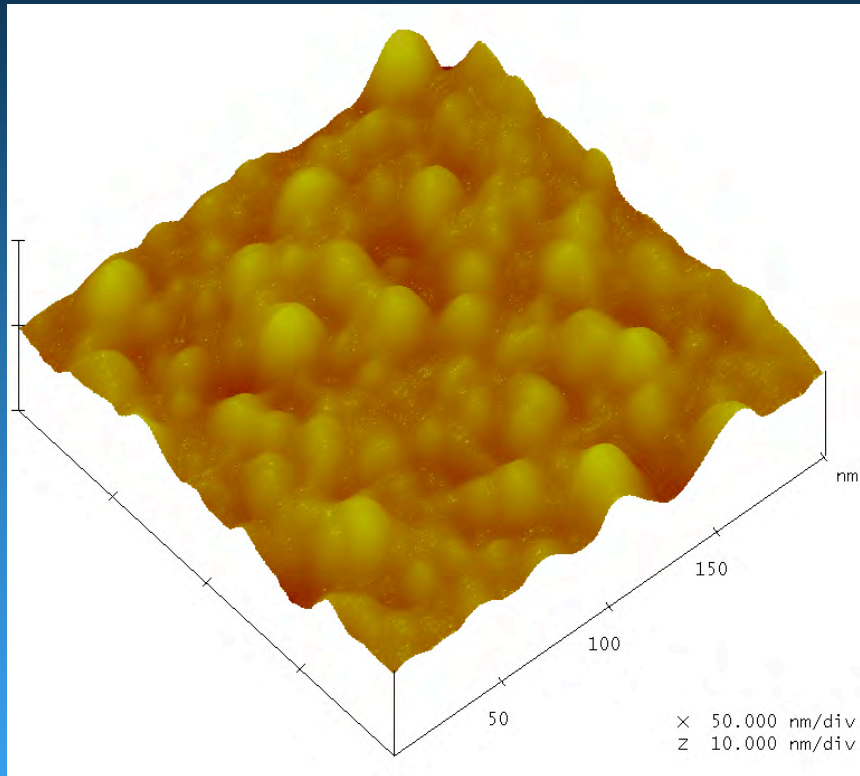
# Amorphous $\text{In}_x\text{O}_y$ Thin Films Exhibiting Local Superconductivity



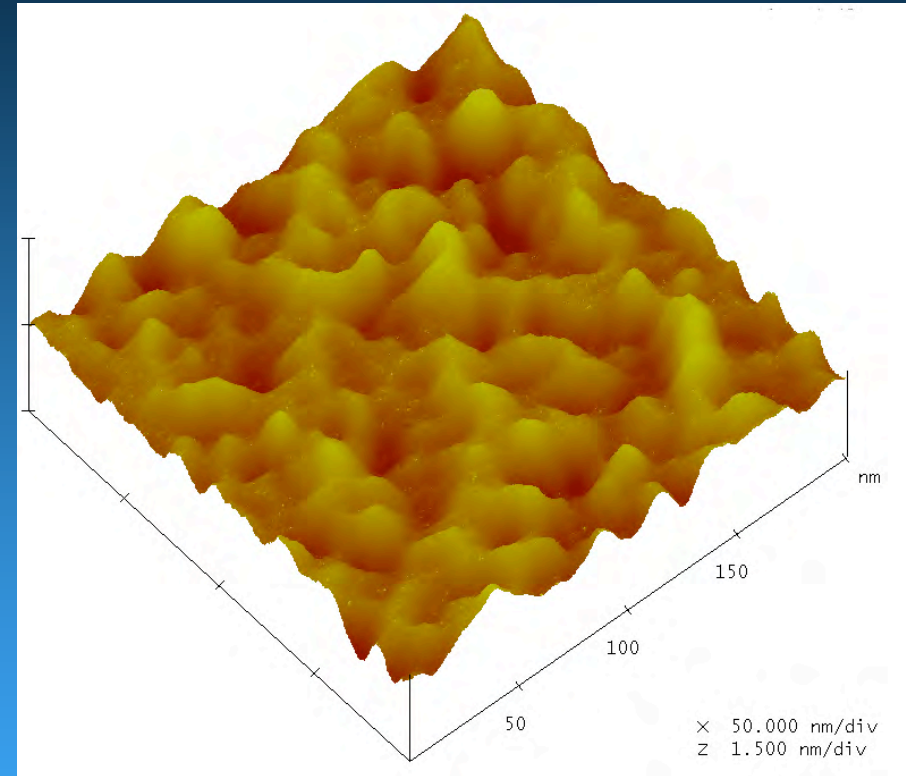
At high temperature the conduction is governed by 3D Mott variable range hopping



# Surface Characterization: AFM

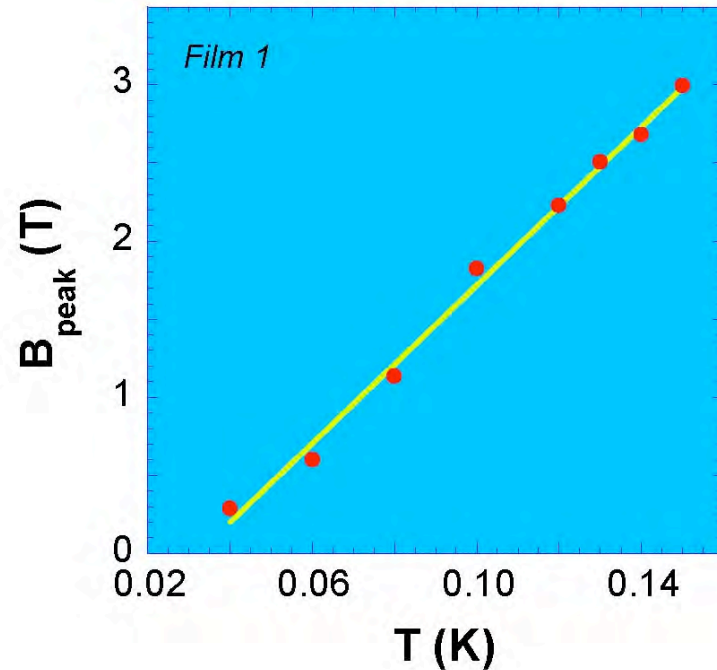
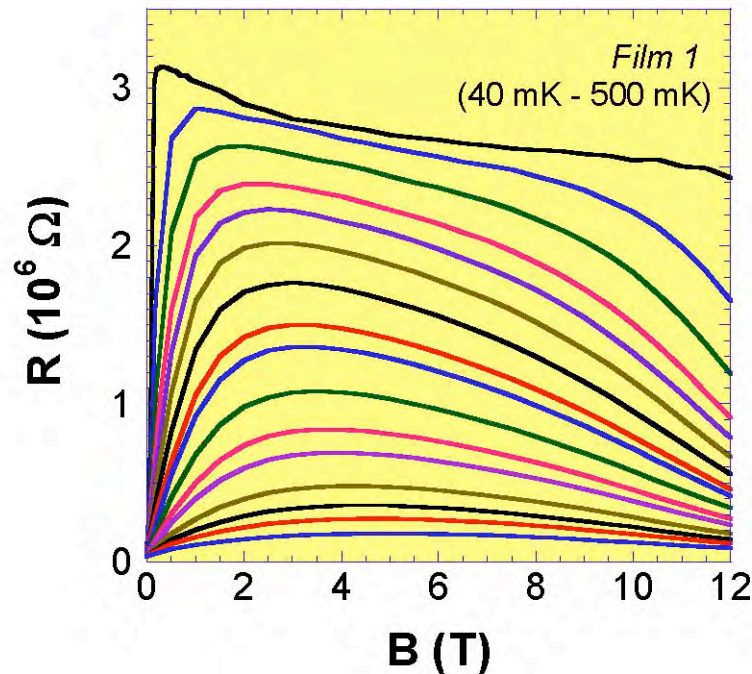


**Insulating Film**  
Max. vertical range:  $85 \text{ \AA}$   
Mean "feature" diameter:  $180 \text{ \AA}$



**Superconducting Film**  
Max. vertical range:  $13 \text{ \AA}$   
Mean "feature" diameter:  $125 \text{ \AA}$

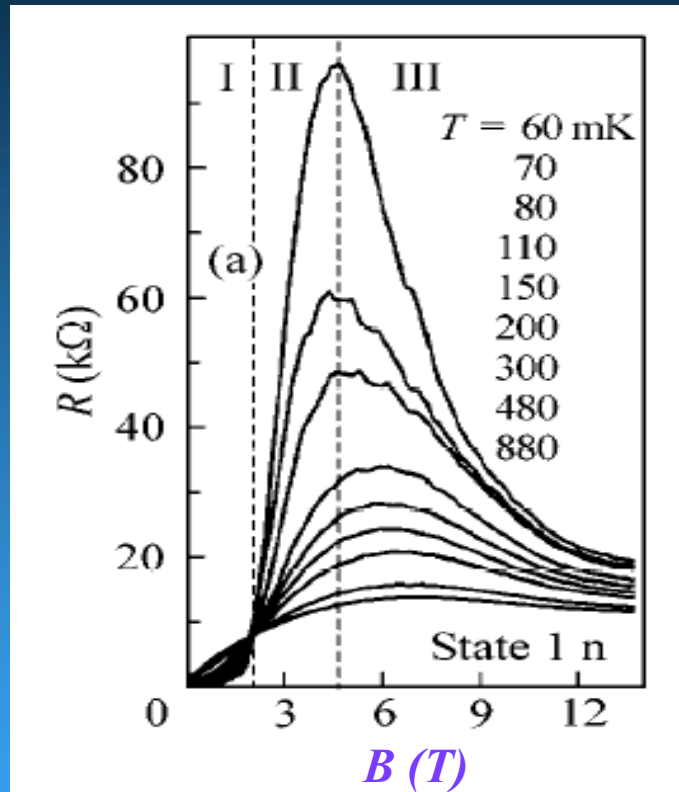
# Evolution of $R(B)$ with Increasing Temperature



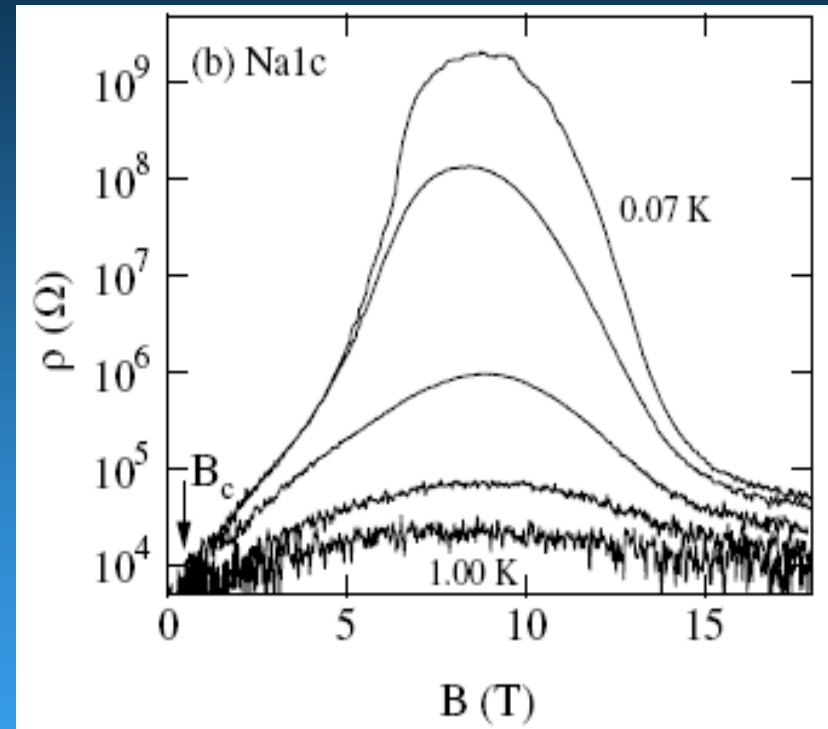
The application of a relatively weak magnetic field results in a dramatic rise in the resistance at low  $T$ .  
( $B = 0.2$  T increases  $R$  by up to a factor of 40 at 40 mK)

Like a granular system, but is not granular.

# Magnetic Field-Tuned SIT in $\alpha$ - $\text{In}_x\text{O}_y$ Films



[Gantmakher, *et al.*, 2000]



[Sambandamurthy, *et al.*, 2004]

**Material, microstructure, disorder, carrier concentration determine the magnitude of the resistance peak**

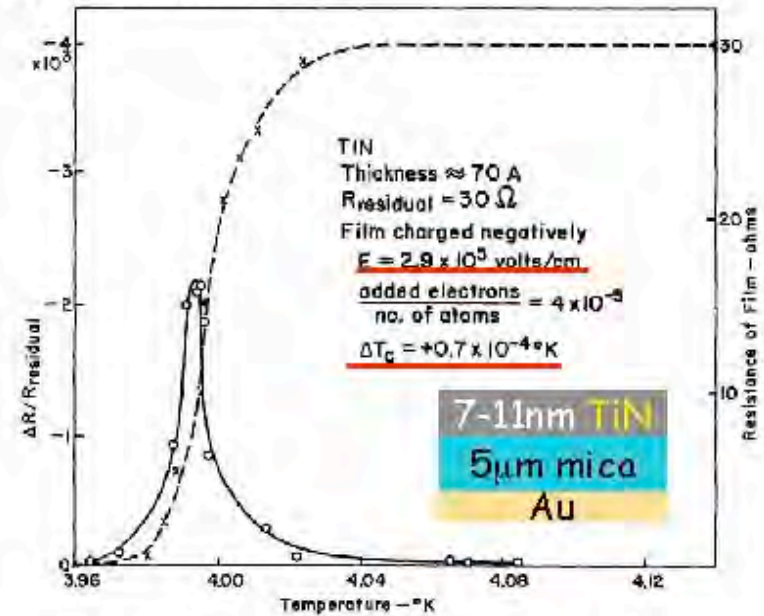
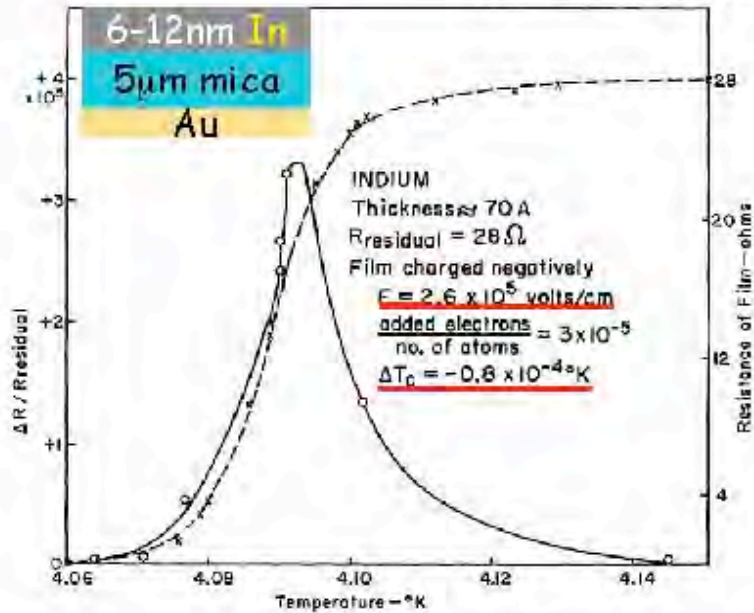
# Control by Electrostatic Charging

Choose systems in which there is a close competition between two or more electronic phases, in which-

small changes in chemical composition, strain, or disorder bring about transitions between phases

Pick chemical composition or disorder placing system near a phase boundary and then alter charge density electrostatically to traverse the boundary.

## CHANGES IN SUPERCONDUCTING CRITICAL TEMPERATURE PRODUCED BY ELECTROSTATIC CHARGING\*

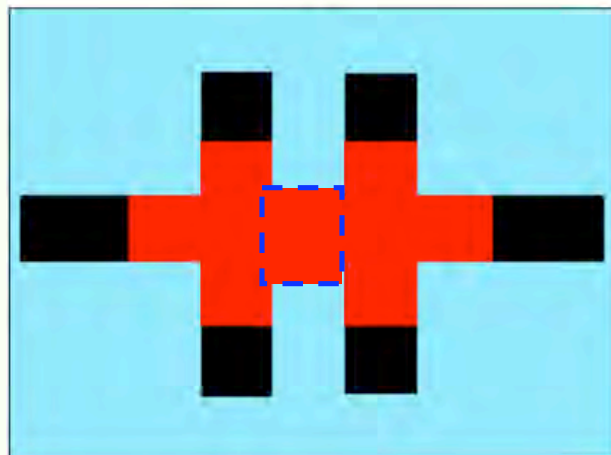
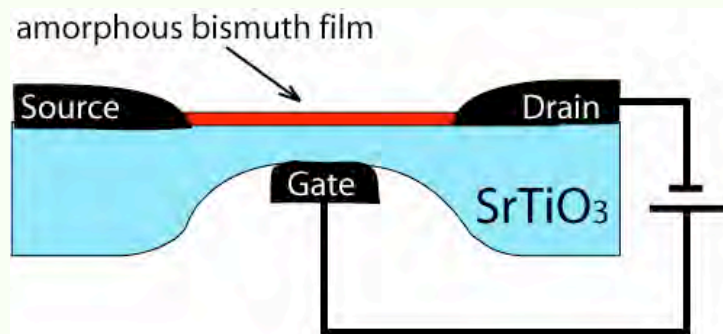
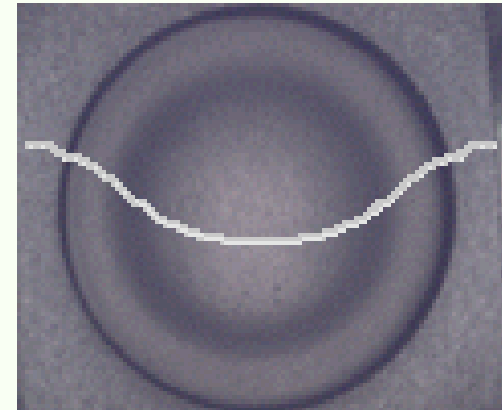


R. E. Glover III and M. D. Sherrill, Phys. Rev. Lett. **5**, 248 (1960)

$$\frac{|\Delta T_c|}{T_c} \approx 0.002\%$$

# FET Structure: Combined Substrate and Gate Insulator

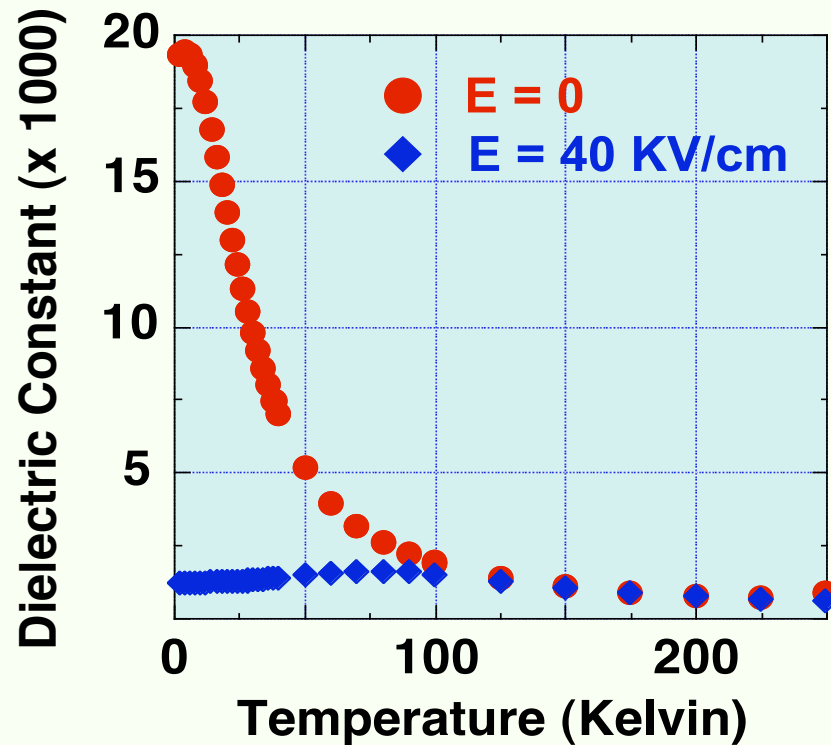
Back of a micro-machined substrate.  
Height profile is superimposed on the picture.  
Thickness in middle can range from  $10\mu\text{m}$  to  $100\mu\text{m}$   
Surface roughness of approximately  $1\mu\text{m}$ .  
Diameter of the thinned region is typically  $4\text{mm}$ .



Cartoon of insulating substrate separating a Bi film from the gate. Thickness of the film is about  $10\text{ \AA}$ , and that of the source and drain about  $100\text{ \AA}$ . Separation between the gate and the film is approximately  $50\text{ }\mu\text{m}$ .

## Why Strontium Titanate?

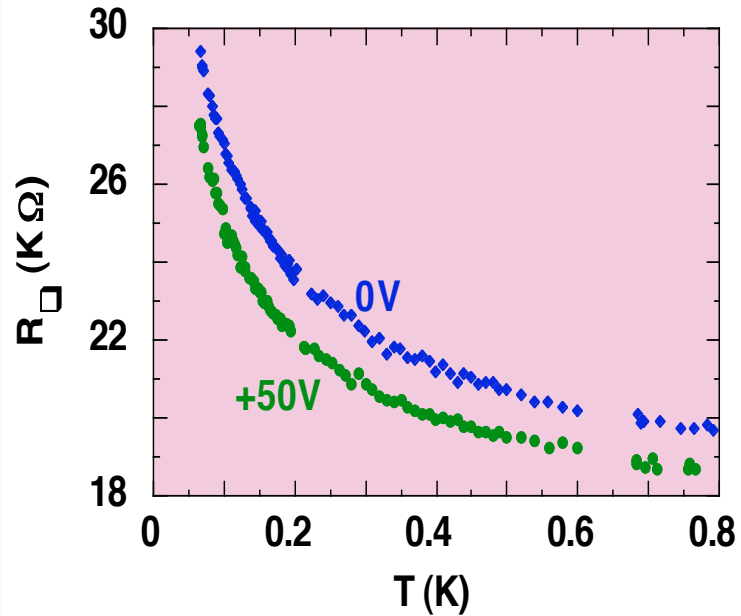
### SrTiO<sub>3</sub> as a Dielectric for Electrostatic Doping



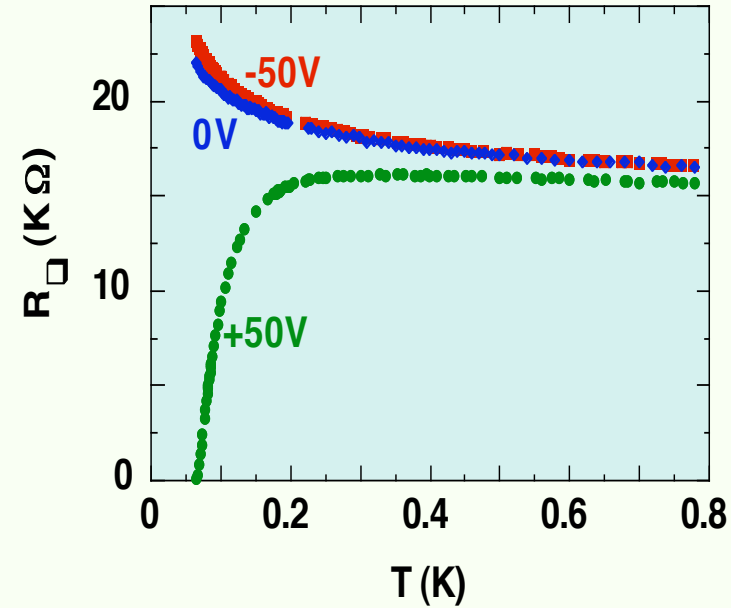
$\kappa_e > 19,000$  below  
10 K

A. Bhattacharya, *et al.*, APL 85, 997 (2004)

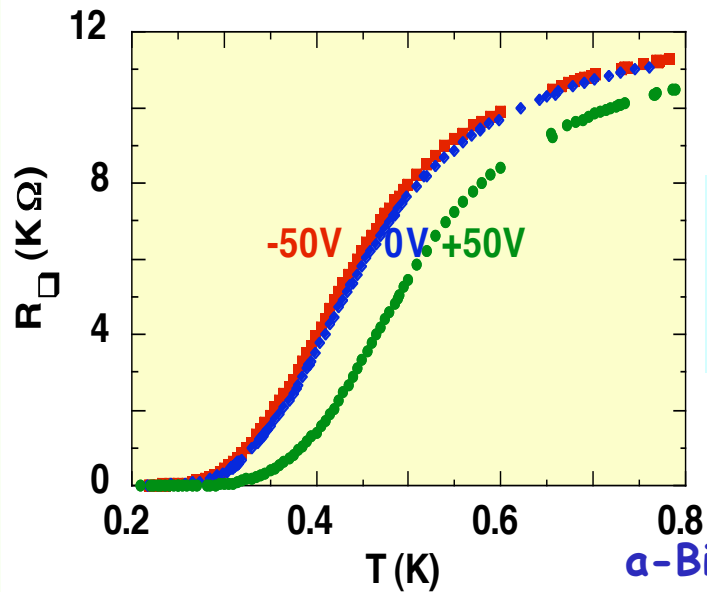
## Electrostatic "doping" at various film thicknesses



$a$ -Bi Film with Thickness 9.91 Å



$a$ -Bi Film with Thickness 10.22 Å



$a$ -Bi Film with Thickness 10.59 Å

$T_{co}$  ( $T_c$  at  $V_g = 0$  volt) = 446 mK

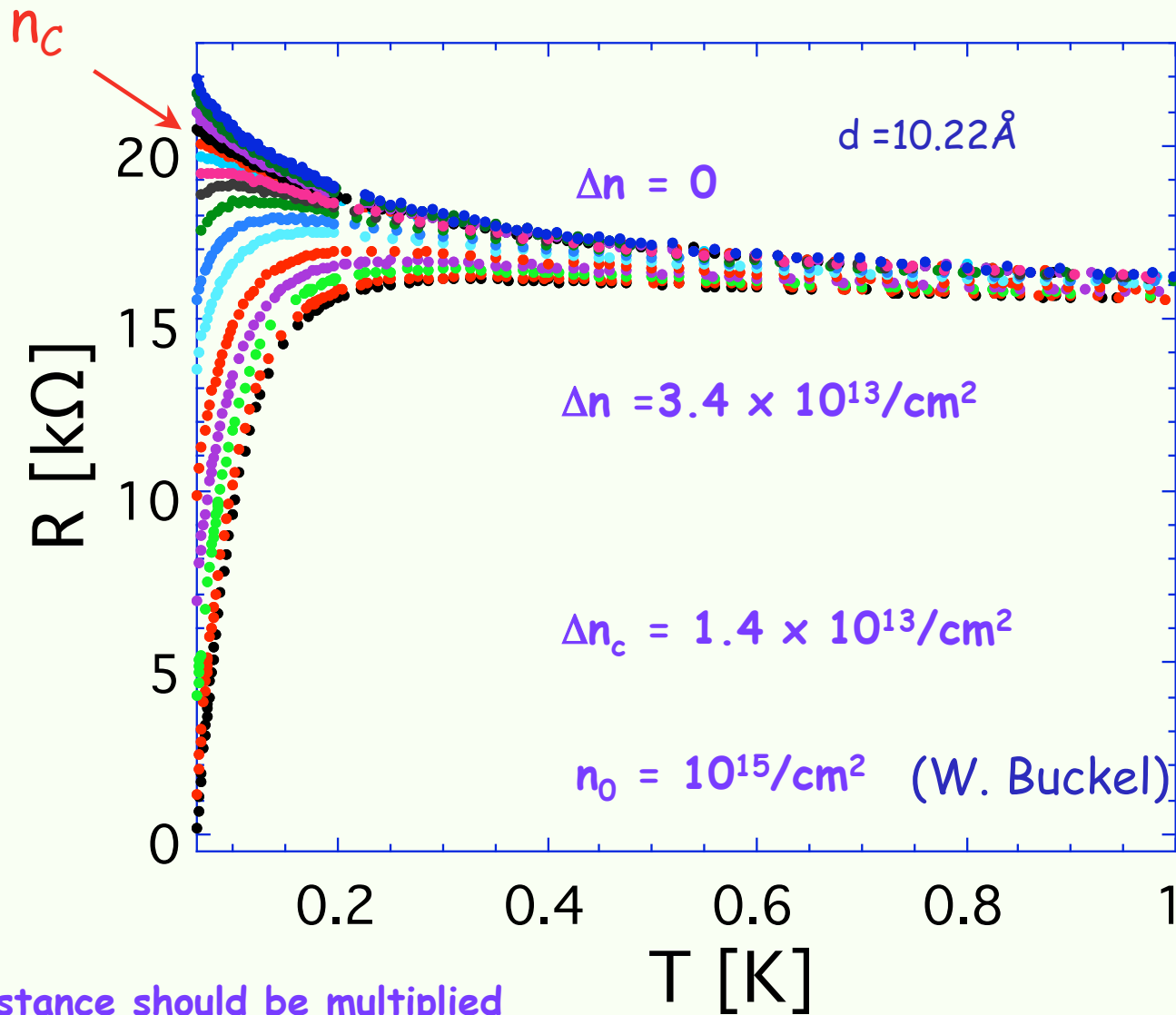
$V_g = 50$  volts increases  $T_c$  by 56 mK

$V_g = -50$  volts decreases  $T_c$  by 10 mK

50 volts is about  $3 \times 10^{13}$  carriers/cm<sup>2</sup>



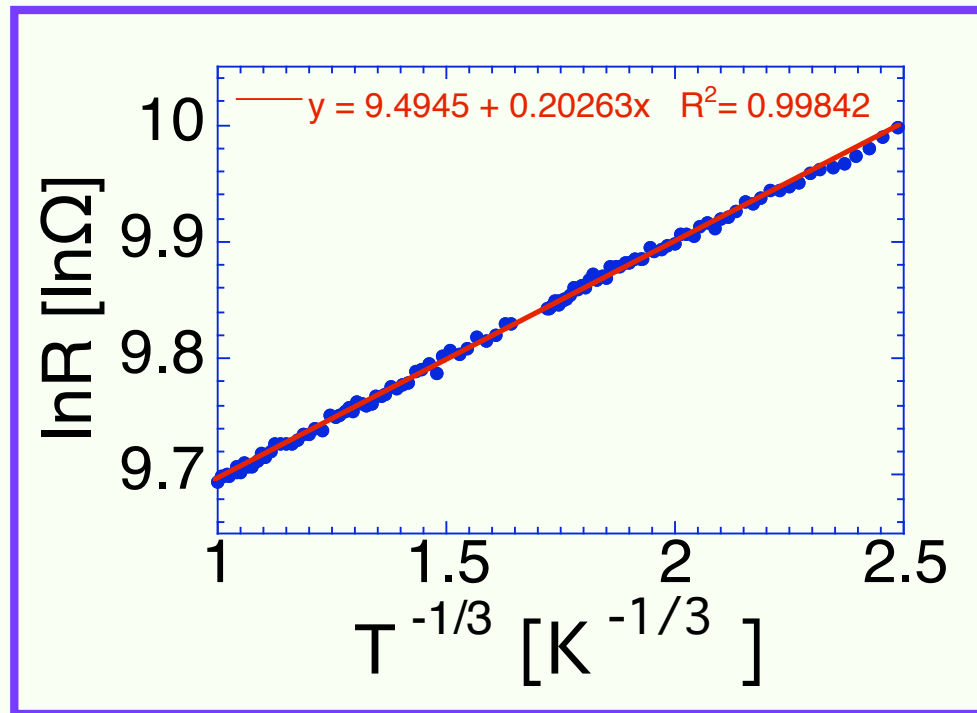
## Electrostatically Tuned S-I Transition



Resistance should be multiplied  
by 0.55

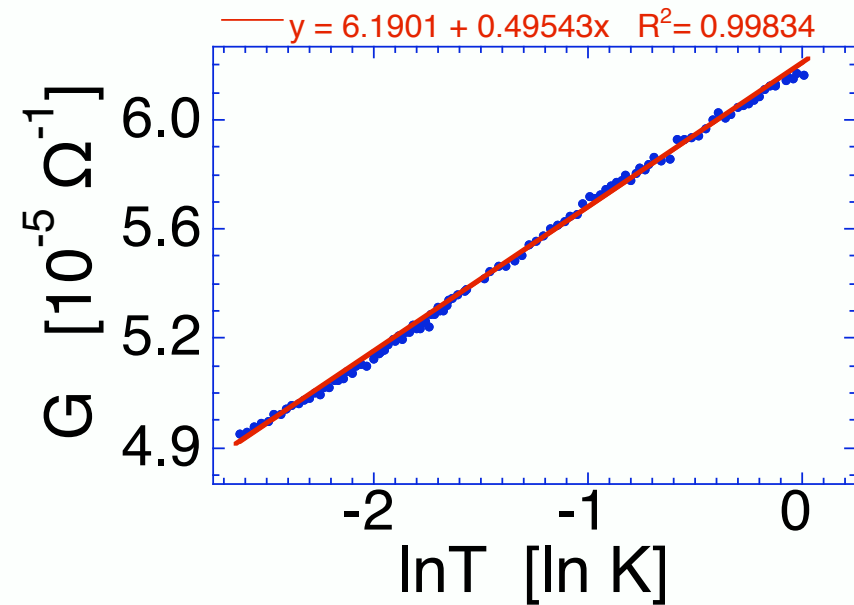
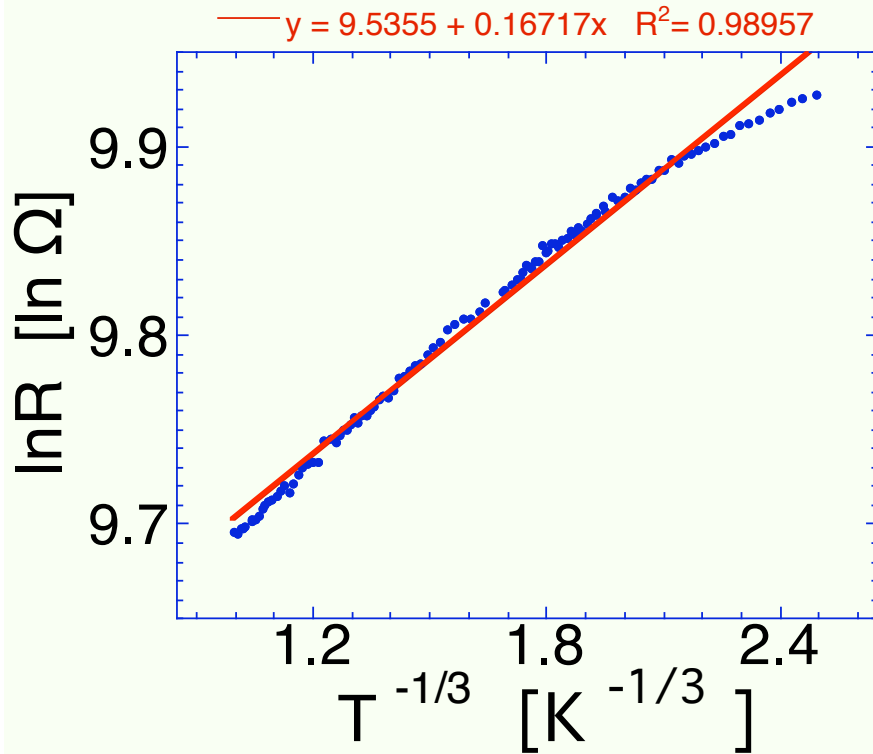
## Systematics of the Insulating State

Mott Hopping Conduction:  $R(T) = R_0 \text{Exp} \left[ \left( \frac{T_0}{T} \right)^{\frac{1}{3}} \right]$



*Strong Screening* due to high dielectric constant  
suppresses the Coulomb Gap

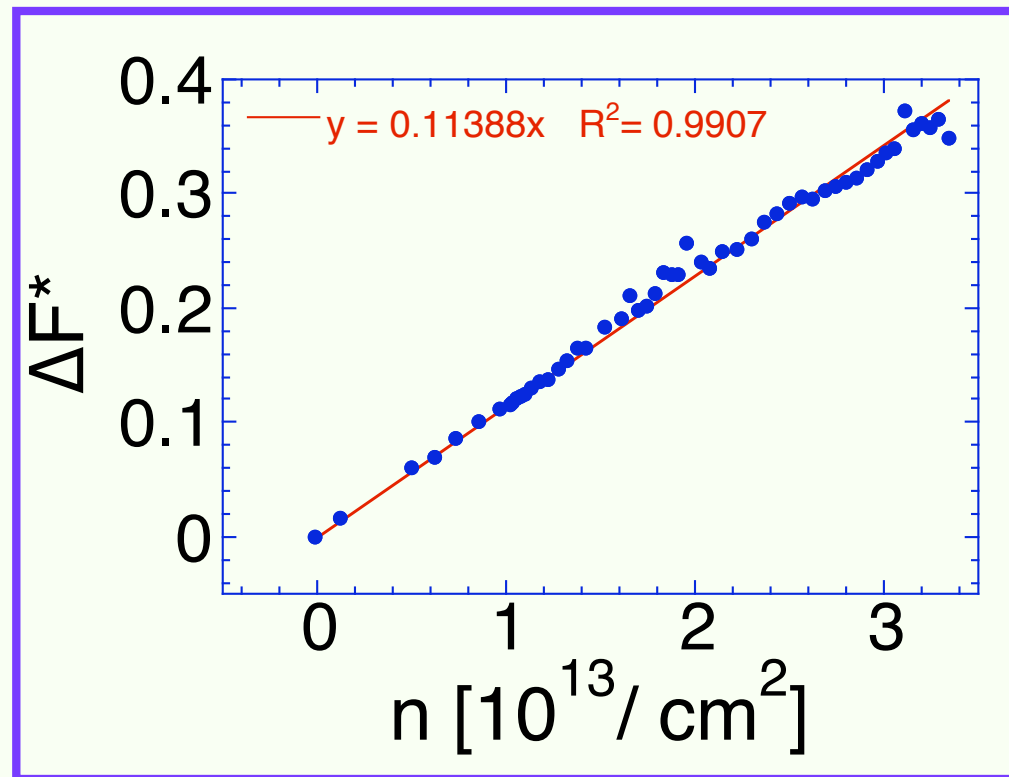
The insulator becomes *weakly localized* precisely when superconductivity appears!



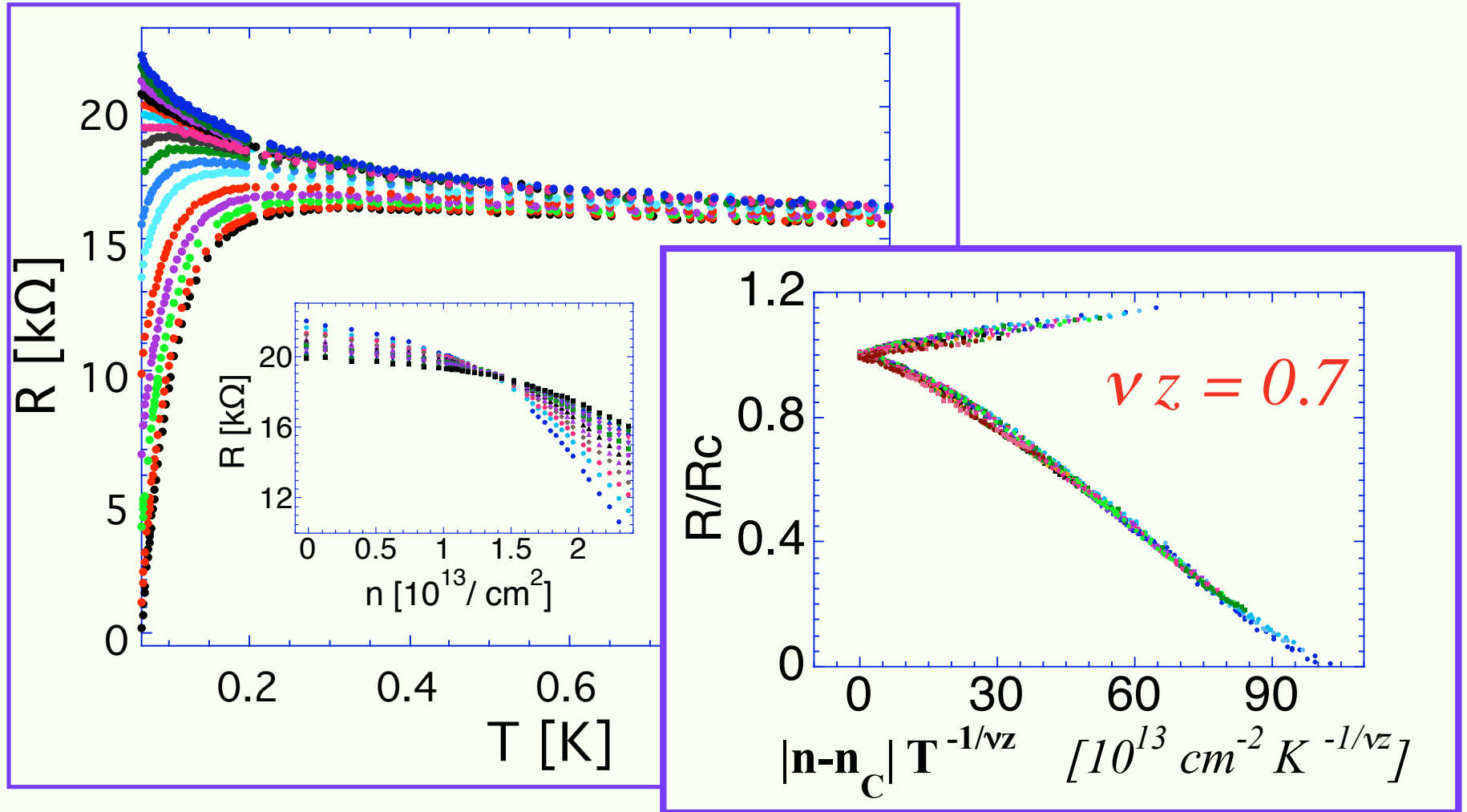
## Both Screening and $N(E_F)$ Influence Superconductivity

Weak Localization and Electron-Electron Interaction Effects?

$$G(T, n) = G_B(n) + \left[ \alpha p - \frac{3}{4} F^*(n) \right] \frac{e^2}{2\pi^2 \hbar} \ln\left(\frac{T}{T_0}\right)$$

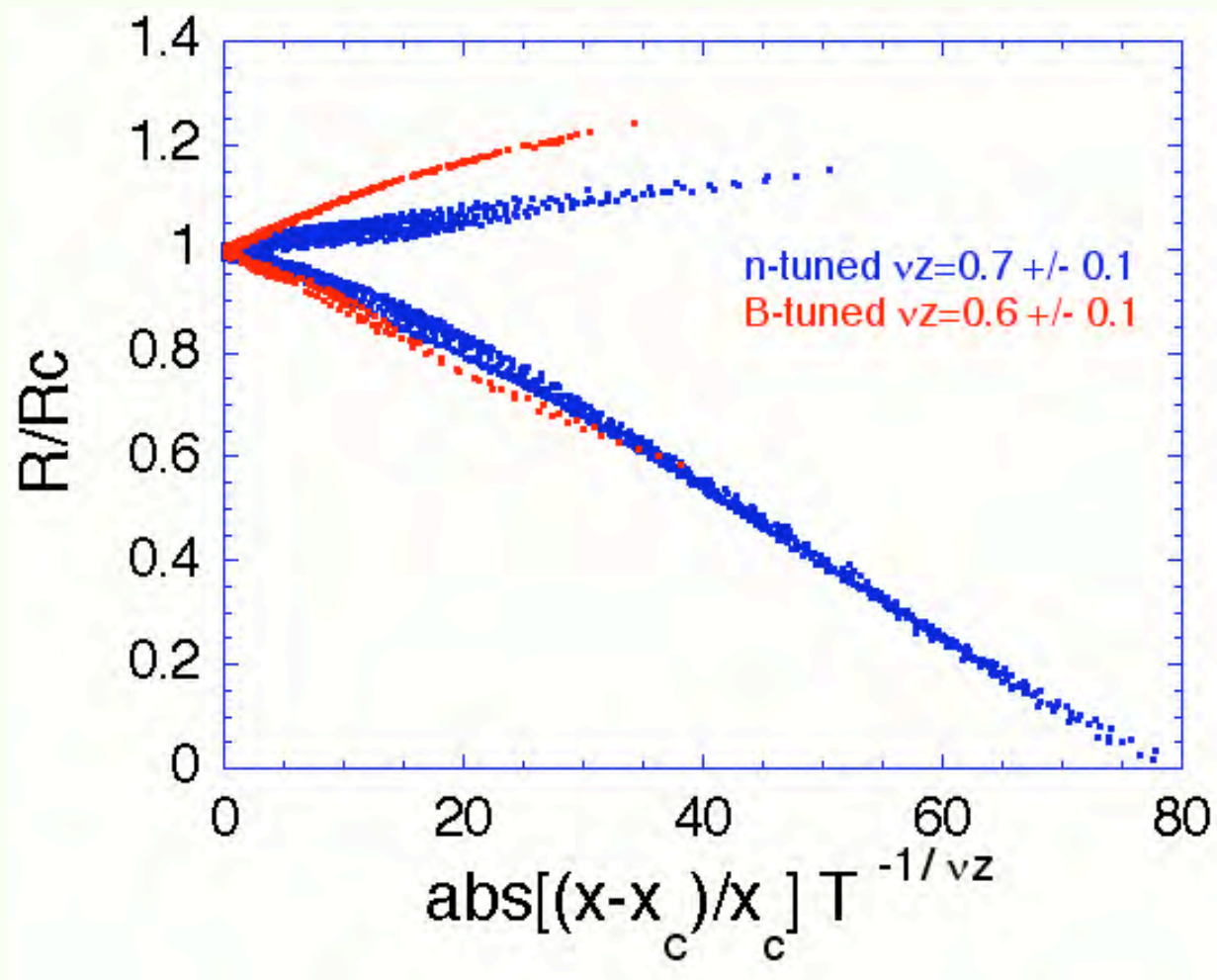


# Scaling



If  $z = 1$  this is universality class of the 2D+1 XY model or Boson Hubbard model  
without disorder

## Scaling plot for n-Tuned and B-Tuned Transitions



Insulating film  $\rightarrow$  superconducting film via the electric field effect

This coincides with a crossover from Mott VRH to  $\ln T$  transport.

The data scale well

$\nu z \sim 0.7 \rightarrow$  universality class. If  $z=1$

2D+1 XY model

or

Boson Hubbard model without disorder

Critical Resistance:  $10,500 \Omega/\square$ . This is not  $6450 \Omega/\square$

Electrostatically induced superconductivity killed by parallel magnetic field - insulating state has a 20% higher resistance at maximum field than original insulator, critical resistance is higher than that of electrostatic tuning case. Same  $\nu z$  product, and normalized superconducting curves seem to overlap.

In-plane electric field scaling does not work because of hot electron effects. It also should not work in most systems where it has been studied.

Electric field effect: K. Pareno *et al.*, PRL 95,049902 (2005)

SrTiO<sub>3</sub>: A. Bhattacharya, *et al.*, APL 85, 997 (2004)

Magnetic field tuned transition, K. Pareno *et al.*, PRB 73, 174527 (2006).

Hot electron effects, K. Pareno *et al.*, PRB 74, 134517 (2006)

## Some Observations

Absence of a universal resistance at threshold. Is it universal in the limit of low disorder?

**Inconsistency of critical exponents:**  $\nu z = 4/3$  is close to value for percolation in 2D for field tuned transitions of  $\text{In}_2\text{O}_3$  and  $\text{Mo}_x\text{Ge}_y$ .

Also found in thickness tuned transitions, *but 0.7 is found for field and charge-tuned transitions in ultra-thin films.*

*Electric Field scaling loses to heating.*

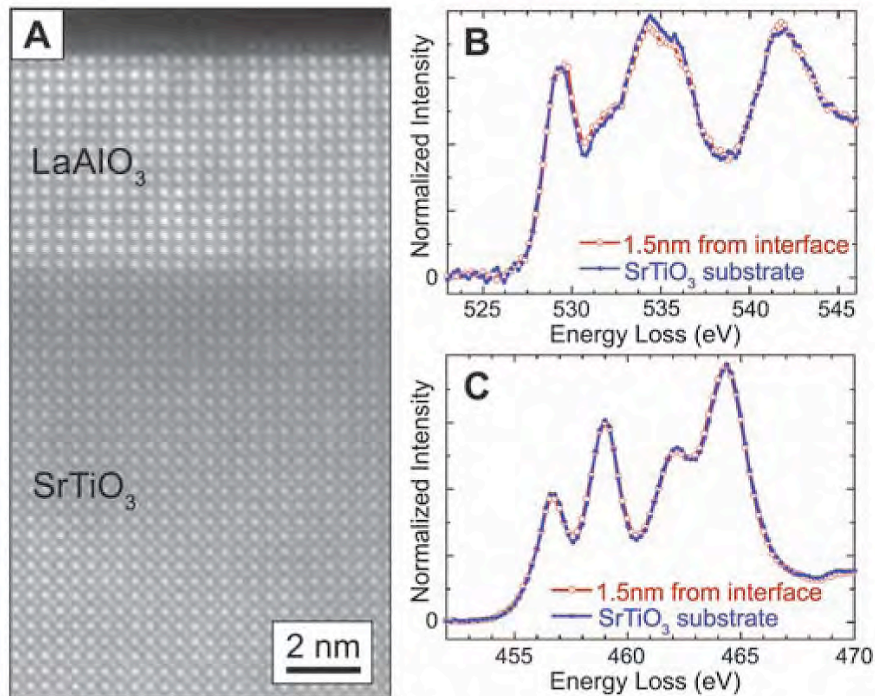
**Metallic Behavior in Field for  $\text{Mo}_x\text{Ge}_y$**  - breakdown of scaling at low T, (Mason and Kapitulnik) followed by hysteretic transition to "true" superconductivity (Mason and Kapitulnik). Not found in quench-evaporated films.

**Giant peak in magnetoresistance in the insulating regime** (Shahar, Kapitulnik, Gantmakher, Hebard and Palaanen, Baturina) system is probably breaking up into puddles as appears to be the case when disorder destroys superconductivity. This may explain the percolation exponent found in many experiments. **This peak is not found in quench-evaporated films.**



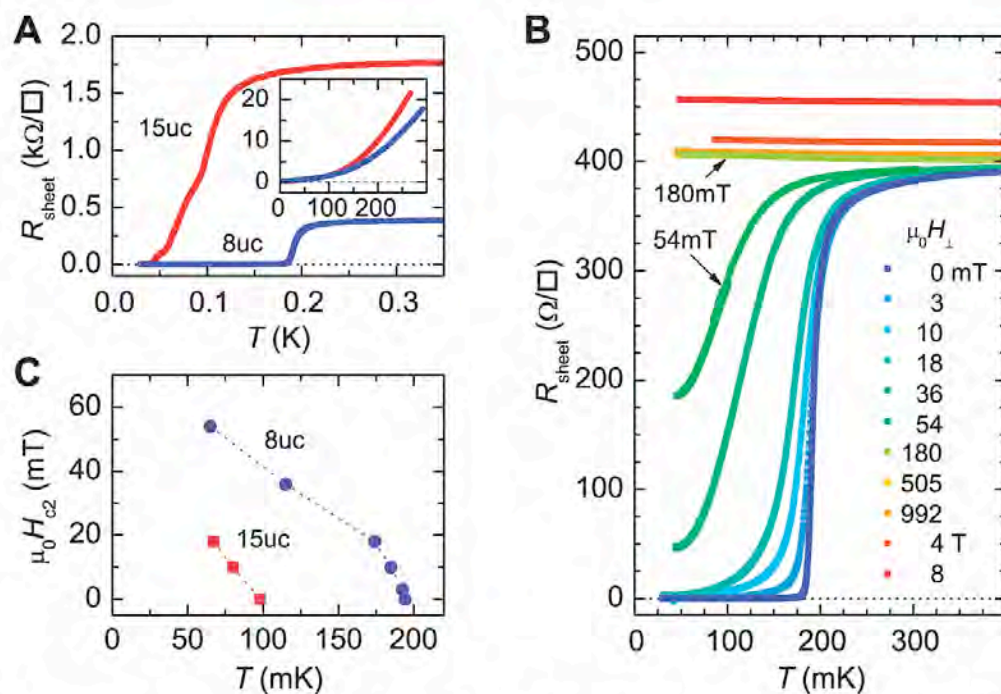
# Possible New Direction: $\text{LaAlO}_3/\text{SrTiO}_3$ Interface Superconductivity

**Fig. 1.** STEM and EELS analysis of a  $\text{LaAlO}_3/\text{SrTiO}_3$  heterostructure. (A) High-angle annular dark field image of a 15-uc-thick  $\text{LaAlO}_3$  film grown on  $\text{SrTiO}_3$  showing a coherent interface. (B) O-K EELS spectra of the  $\text{SrTiO}_3$  close to (1.5 nm) and far away from the interface. Even at 1.5 nm from the interface, the O-K fine structure is only very slightly damped compared with the bulk. The damping could be caused by the presence of a low concentration of oxygen vacancies. (C) Small changes of the Ti-L<sub>2,3</sub> fine structure close to the interface are consistent with a small concentration of  $\text{Ti}^{3+}$ , which falls below the detection limit by 6 nm from the interface and beyond.



N. Reyren, *et al.*, *Science* **317**, 1196 (2007) (Mannhardt group)

## Charge transfer or oxygen defects?



**Fig. 2.** Transport measurements on LaAlO<sub>3</sub>/SrTiO<sub>3</sub> heterostructures. **(A)** Dependence of the sheet resistance on  $T$  of the 8-uc and 15-uc samples (measured with a 100-nA bias current). (Inset) Sheet resistance versus temperature measured between 4 K and 300 K. **(B)** Sheet resistance of the 8-uc sample plotted as a function of  $T$  for magnetic fields applied perpendicular to the interface. **(C)** Temperature dependence of the upper critical field  $H_{c2}$  of the two samples.

Is this really interfacial  
Superconductivity?

Thanks to:

Kevin Parendo--->US Patent Office  
Sarwa Tan

Melissa Eblen-Zayas ---->Carleton College  
Anand Bhattacharya ---->Argonne National Lab.  
Neal Staley ----->Penn State University

Earlier Work

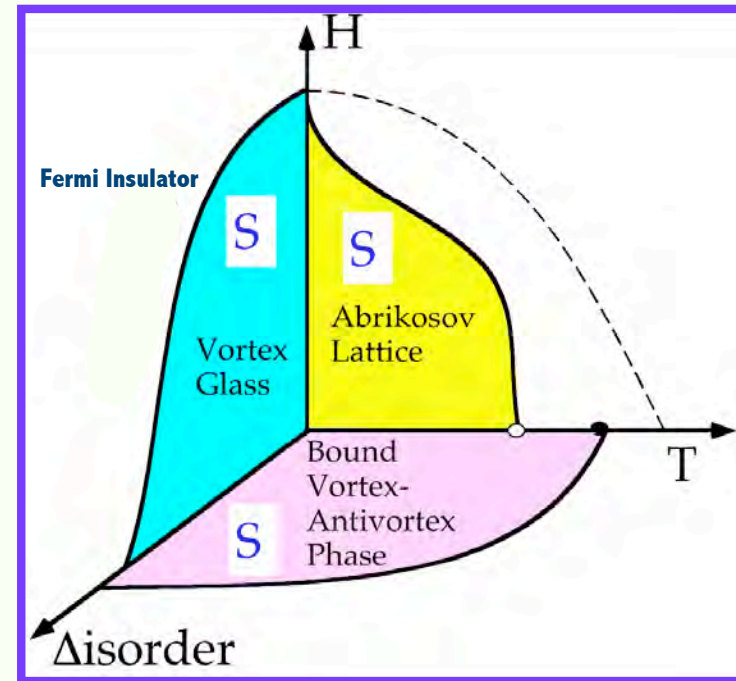
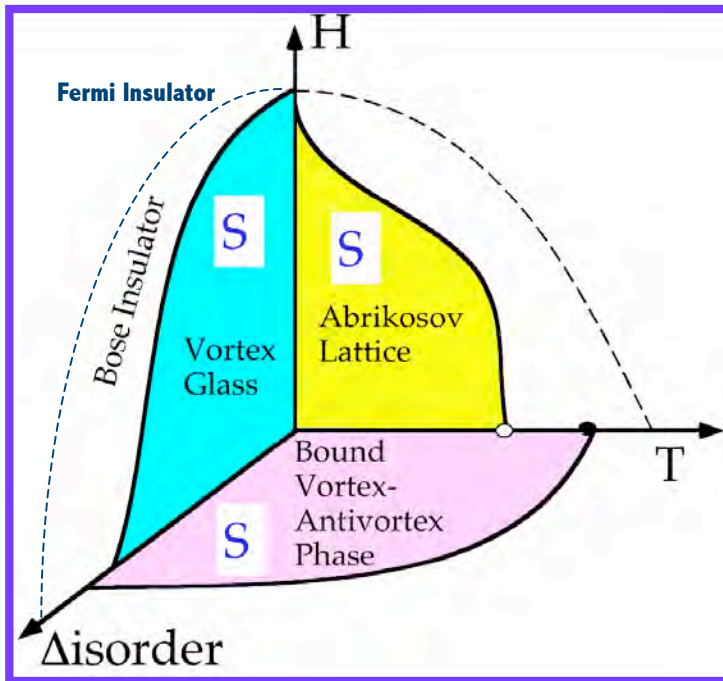
Bradford Orr---->University of Michigan  
David Haviland--->Royal Institute, Stockholm  
Ying Liu----->Penn State University  
Heinrich Jaeger--->University of Chicago

Zvi Ovadyahu, Hebrew University  $\text{In}_2\text{O}_3$  films

# Two Possible Superconductor-Insulator Transitions at $T=0$

Superconductor --> Bose-Insulator

Superconductor --> Fermi-Insulator



Bosons in a random potential. Pairs can become localized due to Coulomb repulsion. Equivalent to array of Josephson-Junctions ( $E_J$  vs.  $E_C$ ).

M.P.A. Fisher, Phys. Rev. Lett. (1990).

No free electrons exist!

Superconductivity is destroyed by disappearance of Cooper pairs altogether. Cooper attraction is reduced due to Large Coulomb interaction.

A.M.Finkelstein, JETP Lett. 45, 46 (1987).

This model however neglects quantum fluctuations of the Bosonic field!

## What is a Quantum Critical Point?

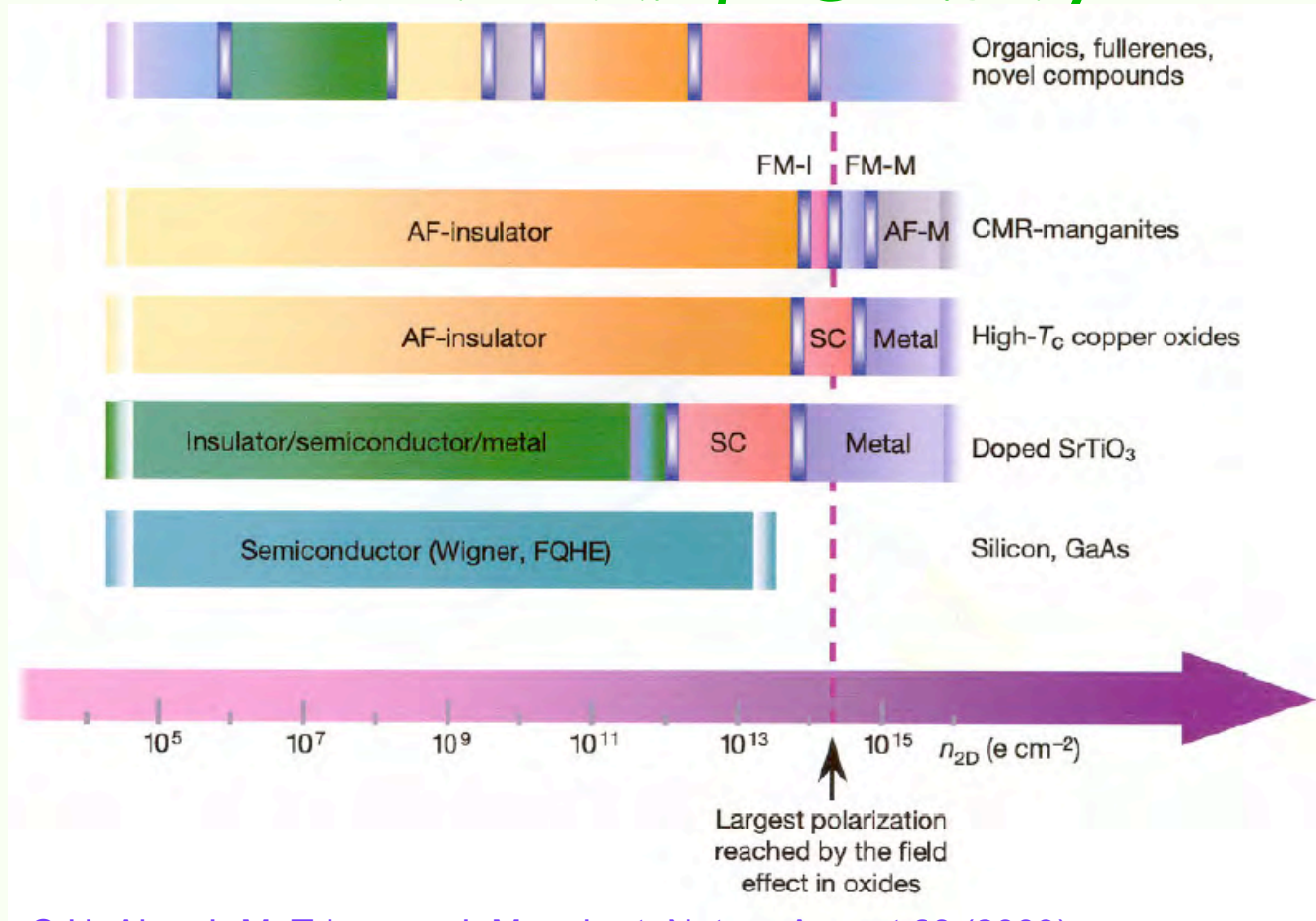
**Classical critical point**-- thermal fluctuations--scale invariance, divergent correlation length. The free energy is a non-analytic function at  $T = T_c$ .

**Quantum critical point**--quantum fluctuations at  $T = 0$  -- scale invariance, divergent correlation lengths. The ground state energy is a non-analytic function of a **tuning parameter** at  $g = g_c$ .

**The tuning parameter** may be charging energy in a Josephson junction array, magnetic field in a superconductor-insulator transition, or a quantum Hall plateau transition, doping (which destroys antiferromagnetism) in the parent compound of a high- $T_c$  superconductor, or disorder in a conductor, which produces a metal-insulator, or superconductor-insulator transition.

**Why is this important?** There is new physics, tied to the uncertainty principle, in the critical regime of a qcp.

# Properties of Materials as a Function of Sheet Charge Density



From: C.H. Ahn, J.-M. Triscone, J. Mannhart, Nature August 28 (2003).

Also see: Ahn *et al.* Rev. Mod. Phys. (2006).