Origins of the Theory of Superconductivity

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Theoretical remark on the superconductivity of metals*

A. Einstein
translated by Bjoern S. Schmekel (Cornell University)†

The theoretical oriented scientist cannot be envied, because nature, i.e. the experiment, is a relentless and not very friendly judge of his work. In the best case scenario it only says “maybe” to a theory, but never “yes” and in most cases “no”. If an experiment agrees with theory it means “perhaps” for the latter. If it does not agree it means “no”. Almost any theory will experience a “no” at one point in time - most theories very soon after they have been developed. In this paper we want to focus on the fate of theories concerning metallic conductivity and on the revolutionary influence which the discovery of superconductivity must have on our ideas of metallic conductivity.

After it had been recognized that negative electricity is caused by subatomic carriers of particular mass and charge (electrons), there were good reasons to believe that metallic conductivity rests on the motion of electrons. Furthermore, the fact that heat is conducted much better by metals than by non-metals as well as the Wiedemann-Franz law about the substance-independence of the ratio of electric and thermal conductivity of pure metals (at room temperature) led to attribute the thermal conductivity to electrons as well. Under these circumstances there were reasons for an electron-based theory of metals similar to the kinetic gas theory (Riecke, Drude, H. A. Lorentz). In this theory free electron motion is assumed which resembles gas molecules with thermal mean kinetic energy $3/2 kT$ neglecting collisions.

*from “Gedenkboek aangeb. aan H. Kamerlingh Onnes, e.a. Leiden, E. J.Jo, 1922, pp. 435” / translated with courtesy of the Kamerlingh Onnes Laboratory, Leiden - Institute of Physics, Leiden University
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The Simple Facts of Superconductivity (as of 1955)
In 1911, Kammerling Onnes found that the resistance of a mercury sample disappeared suddenly below a critical temperature.
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$$T > T_c$$

$$T < T_c$$
Further, the electronic specific heat increases discontinuously at $T_c$ and vanishes exponentially near $T=0$. 
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This and other evidence, indicates the existence of an energy gap in the single particle electronic energy spectrum.
And it has been recently discovered that the transition temperature varies with the mass of the ionic lattice as
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$$\sqrt{M} \ T_c = \text{constant}$$
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$$\sqrt{M} \ T_c = \text{constant}$$

This is known as the isotope effect and indicates that the electron-phonon interaction is implicated in the transition into the superconducting state.
Discussing these same phenomena today we would be obligated to say that
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Type I superconductors show a Meissner effect, while type II superconductors do not.

Some superconductors show a specific heat curve such as the one above, while others do not.

Some superconductors seem to have no energy gap and others show no isotope effect.
Yet the 1955 simple facts of superconductivity capture the great qualitative change that occurs in the transition to the superconducting state.
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By focusing our attention on them we were able to construct a theory of superconductivity.
The Sommerfeld-Bloch individual particle model (refined as Landau’s Fermi liquid theory) gives a fairly good description of normal metals but no hint of superconductivity.
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\[ k_F \approx 10^8 \text{ cm}^{-1} \]

\[ E_F \approx 5 \text{ eV} \]
The Sommerfeld-Bloch individual particle model (refined as Landau’s Fermi liquid theory) gives a fairly good description of normal metals but no hint of superconductivity.

The normal ground state wavefunction is a filled Fermi sphere for both spin directions.

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\[ E_F \approx 5 \text{ eV} \]
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Thus, the qualitative changes that occur at the transition temperature involve an energy change orders of magnitude smaller than the Coulomb energy.

This huge energy difference contributed to the great difficulty of the problem.
But the Coulomb interaction is present in every metal; some of them are superconductors, and some are not, so one is led to guess that this interaction, even with its large energy, is somehow irrelevant.
But the Coulomb interaction is present in every metal; some of them are superconductors, and some are not, so one is led to guess that this interaction, even with its large energy, is somehow irrelevant.

After the work of Fröhlich, Bardeen and Pines the interaction that produces superconductivity was believed to arise due to phonon exchange which, under some conditions, would be an attractive interaction between electrons near the Fermi surface.
Normally phonons produce resistance
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But the exchange of phonons can produce an interaction between electrons:
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This interaction is attractive in a small region near the Fermi surface and may in some cases dominate the repulsive Coulomb interaction.
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The density of states of one spin at the Fermi surface, $N(0) \approx \frac{10^{22} \text{ levels}}{\text{eV cm}^3}$.
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The system is very degenerate.
The problem becomes how to diagonalize such a highly degenerate matrix.
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Coherent superpositions of the original states, that are separated from them by a volume independent energy gap, can appear.
In the many electron system one can pick out highly degenerate submatrices in which a pair of electrons scatter near the Fermi surface.
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Every such singlet spin zero-momentum pair state can scatter to every other.
Diagonalizing one such pair submatrix gives the surprising result
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\[ 2E_F \quad 2(E_F + \hbar \omega) \]

\[ E \rightarrow \]
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A single state, which is a coherent superposition of the original states, is separated from the rest by a volume independent energy gap.
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A single state, which is a coherent superposition of the original states, is separated from the rest by a volume independent energy gap.

\[ 2E_F \quad \text{to} \quad 2(E_F + \hbar \omega) \]
Allow me to show you a page from my notes of that period with the pair solutions as they first appeared to me.
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The last crossing on the left is the coherent state.
It is split from the continuum by a volume independent energy gap, displaying, the now well-known, essential singularity in the coupling constant.

Allow me to show you a page from my notes of that period with the pair solutions as they first appeared to me.

The last crossing on the left is the coherent state.

It is split from the continuum by a volume independent energy gap, displaying, the now well-known, essential singularity in the coupling constant.
\[ E - 2E_F = \frac{-2\hbar \omega}{2 \sqrt{N(0)V \ln e - 1}} \]
In the weak coupling limit \( N(0)V \ll 1 \) this becomes

\[
E - 2E_F = \frac{-2\hbar\omega}{2\sqrt{N(0)V}} e^{-1}
\]
In the weak coupling limit \( N(0)V \ll 1 \) this becomes

\[
E - 2E_F \approx -2(\hbar \omega) e^{-\frac{2}{N(0)V}}
\]
The energy of a ground state composed of such “bound” pair states would be proportional to \((\hbar \omega)^2\) and therefore inversely proportional to the isotopic mass as expected.
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In addition, the exponential factor seemed to give a natural explanation of why the transition energy into the superconducting state is so small.
However in the actual situation the pairs overlap.
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One can estimate that $10^6$ to $10^7$ pairs occupy the same volume.
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BCS solves this problem by introducing a wave function that satisfies the Pauli exclusion principle and that contains the many overlapping non-interacting pairs.
The ground state wave function of the BCS superconductor is a linear superposition of states in which pairs \((k \uparrow - k \downarrow)\) are occupied or unoccupied.
The ground state wave function of the BCS superconductor is a linear superposition of states in which pairs \((k \uparrow \rightarrow -k \downarrow)\) are occupied or unoccupied.
The BCS ground state wave function can also be regarded as the anti-symmetrized product of single pair wave functions of the form $\chi^{\uparrow\downarrow}(r_1 - r_2)$.
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$$\psi_0 = A[\chi_{\uparrow \downarrow}(r_1 - r_2)\chi_{\uparrow \downarrow}(r_3 - r_4)\cdots\chi_{\uparrow \downarrow}(r_{N-1} - r_N)]$$
The BCS ground state wave function can also be regarded as the anti-symmetrized product of single pair wave functions of the form $\chi_{\uparrow\downarrow}(r_{1} - r_{2})$.

$$\psi_0 = A[\chi_{\uparrow\downarrow}(r_{1} - r_{2})\chi_{\uparrow\downarrow}(r_{3} - r_{4})...\chi_{\uparrow\downarrow}(r_{N-1} - r_{N})]$$

Typically the pair function, $\chi_{\uparrow\downarrow}(r)$, extends $\approx 10^{-4}\text{cm}$; this has come to be called off-diagonal long range order.
The BCS pairing Hamiltonian is composed of a kinetic energy term and an interaction term in which any occupied pair can scatter into any available pair state.
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The ground state is obtained by finding the pair function that minimizes the energy of the pairing Hamiltonian.
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The ground state is obtained by finding the pair function that minimizes the energy of the pairing Hamiltonian.

\[ \langle \psi_0 | H_{\text{pairing}} | \psi_0 \rangle \]
The energy difference between the superconducting and the normal state is

\[ W_n - W_s = \frac{2N(0)(\hbar \omega)^2}{e^{\frac{2}{N(0)V}} - 1} \]
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\[ W_n - W_s = \frac{2N(0)(\hbar \omega)^2}{2^{\sqrt{N(0)V}} e^{N(0)V} - 1} \]

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The energy difference between the superconducting and the normal state is

\[ W_n - W_s = \frac{2N(0)(\hbar \omega)^2}{2 \sqrt{N(0)} e^{\sqrt{N(0)V}} - 1} \]

In the weak coupling limit, this becomes

\[ W_n - W_s = 2N(0)(\hbar \omega)^2 e^{-\sqrt{N(0)V}} \]

The energy gap is

\[ \Delta = 2(\hbar \omega)e^{-\frac{1}{\sqrt{N(0)V}}} \]
The idealized BCS theory reproduces the “simple” facts of superconductivity:
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Zero resistivity
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- Meissner effect
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- Zero resistivity
- Meissner effect
- Specific heat
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Zero resistivity
Meissner effect
Specific heat
Isotope effect
The idealized BCS theory reproduces the “simple” facts of superconductivity:

- Zero resistivity
- Meissner effect
- Specific heat
- Isotope effect
- The energy gap
But, in addition, there are new, subtle and surprising effects.
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In calculating matrix elements of interactions that involve operators such as
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\[ B = \sum_{kk'} B_{kk'} (c_{k'}^\uparrow c_k^\uparrow \pm c_{-k'}^\uparrow c_{-k}^\uparrow) \]
But, in addition, there are new, subtle and surprising effects.

In calculating matrix elements of interactions that involve operators such as

\[ B = \sum_{kk'} B_{kk'} (c_{k'}^* c_k^\uparrow \pm c_{-k'}^* c_{-k}^\uparrow) \]

the initial and final states are connected in a new and unexpected way:
The initial and final states are connected by $c_{k'}^{*} \uparrow c_{k}^{\uparrow}$.
The initial and final states are connected by \( c_{k'}^* c_k \).
The initial and final states are connected by $c_{k'}^* c_k^\uparrow$. 
They are also connected by $c_{-k'}^* c_{-k} \downarrow \downarrow$. 
They are also connected by $c^*_{-k'} c_{-k}$. 

\[ \begin{align*}
  & c^*_{-k'} c_{-k} \\
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\end{align*} \]
They are also connected by $c_{-k'}^* c_{-k'}$.
These occur in calculations of the interaction of superconductors with the electromagnetic field or with phonons:
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Meissner effect
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- Meissner effect
- Ultrasonic attenuation
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- Meissner effect
- Ultrasonic attenuation
- Nuclear spin relaxation
Some of the calculations can become very complex.
Some of the calculations can become very complex.

### Table II. Matrix elements of single-particle scattering operator.

<table>
<thead>
<tr>
<th>Wave functions</th>
<th>Initial, (\Psi_i)</th>
<th>Final, (\Psi_f)</th>
<th>Ground or excited ((\pm))</th>
<th>Energy difference (W_i - W_f)</th>
<th>Probability of initial state</th>
<th>Matrix elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>X0 00 00 X0</td>
<td>X0</td>
<td>+ +</td>
<td>(E - E')</td>
<td>(\frac{1}{2}s(1-s' - p'))</td>
<td>(\frac{1}{2}s(1-s'-p'))</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(\frac{1}{2}s(1-s'-p'))</td>
</tr>
<tr>
<td>(b)</td>
<td>XX 0X 0X XX</td>
<td>XX</td>
<td>+ +</td>
<td>(E - E')</td>
<td>(\frac{1}{2}s'(1-s - p))</td>
<td>(\frac{1}{2}s'(1-s - p))</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(\frac{1}{2}s'(1-s - p))</td>
</tr>
<tr>
<td>(c)</td>
<td>X0 0X 00 XX</td>
<td>XX</td>
<td>+ +</td>
<td>(E + E')</td>
<td>(\frac{1}{2}s's')</td>
<td>(\frac{1}{2}s's')</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(\frac{1}{2}s's')</td>
</tr>
<tr>
<td>(d)</td>
<td>XX 00 0X X0</td>
<td>0X</td>
<td>+ +</td>
<td>(-E + E')</td>
<td>((1-s - p)(1-s' - p'))</td>
<td>((1-s - p)(1-s' - p'))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>X0</td>
<td></td>
<td></td>
<td></td>
<td>((1-s - p)(1-s' - p'))</td>
</tr>
</tbody>
</table>

* For transitions which change spin, reverse designations of \((k', -k')\) in the initial and in the final states.
The superposition of these two amplitudes results in strikingly different behavior for interactions depending on how they transform under time reversal.
In time reversal the coordinates transform as

\[ x \rightarrow x \]
\[ y \rightarrow y \]
\[ z \rightarrow z \]
\[ t \rightarrow -t \]
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\[ x \rightarrow x \]
\[ y \rightarrow y \]
\[ z \rightarrow z \]
\[ t \rightarrow -t \]

while various operators transform as

\[ j(r,t) \rightarrow -j(r,-t) \]
\[ \rho(r,t) \rightarrow +\rho(r,-t) \]
\[ \sigma_z(t) \rightarrow -\sigma_z(-t) \]
Ultrasonic attenuation involves $\rho$
Ultrasonic attenuation involves $\rho$

Nuclear spin relaxation involves $\sigma$
Ultrasonic attenuation, as a function of temperature, falls off very rapidly across the superconducting transition.
Ultrasonic attenuation, as a function of temperature, falls off very rapidly across the superconducting transition.

Morse and Bohm (1957)
In contrast, the nuclear spin relaxation rate increases across the transition temperature.
In contrast, the nuclear spin relaxation rate increases across the transition temperature.

The circles are the experimental data of Hebel and Slichter (1959), the crosses data of Redfield and Anderson (1959).
Comparison with the BCS theory
Comparison with the BCS theory

Ultrasonic Attenuation

Data from Morse and Bohm (1957)
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Ultrasonic Attenuation

Nuclear Spin Relaxation

The circles are the experimental data of Hebel and Slichter (1959), the crosses data of Redfield and Anderson (1959).

Data from Morse and Bohm (1957)
“...such a striking phenomenon as superconductivity [was] ... nothing more exciting than a footling small interaction between atoms and lattice vibrations.”
Since 1957 the situation has become richer and much more complex.
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There have been many new discoveries as well as a myriad of practical applications.
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There have been many new discoveries as well as a myriad of practical applications.

I can mention just a few:
The BCS theory has had a profound effect on other areas of physics.
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Pairing in:
The BCS theory has had a profound effect on other areas of physics.

Pairing in:

Nuclei
The BCS theory has had a profound effect on other areas of physics.

Pairing in:

Nuclei

Neutron stars
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Pairing in:

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Neutron stars
Helium 3
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Pairing in:

- Nuclei
- Neutron stars
- Helium 3

Color superconductivity in dense quark matter
Transition Between Bose Einstein and BCS Condensation
The interaction between Fermions in Bose-Einstein condensates can be varied, using the Feshbach resonance, changing the range of the pair wave function.
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\[ \chi_{\uparrow \downarrow} (r_1 - r_2) \]
The interaction between Fermions in Bose-Einstein condensates can be varied, using the Feshbach resonance, changing the range of the pair wave function

\[ \chi^{\uparrow \downarrow}(r_1 - r_2) \]

so that a transition between a Bose Einstein and a BCS condensation can be seen.
The broken symmetry displayed by the BCS pair function (the order parameter of the Ginsburg-Landau theory)
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\[ \langle 0 | \psi_\downarrow(r) \psi_\uparrow(r) | 0 \rangle \neq 0 \]
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is the model for the Higgs field broken symmetry of the standard model of elementary particles and fields.
The broken symmetry displayed by the BCS pair function (the order parameter of the Ginsburg-Landau theory)

\[ \langle 0 | \psi_{\downarrow}(r)\psi_{\uparrow}(r) | 0 \rangle \neq 0 \]

is the model for the Higgs field broken symmetry of the standard model of elementary particles and fields

\[ \langle 0 | \phi(x) | 0 \rangle \neq 0 \]
Type II Superconductors
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In some situations, it is energetically favorable for a magnetic field to partially penetrate a superconductor breaking it up into many superconducting and normal regions.
Type II Superconductors

In some situations, it is energetically favorable for a magnetic field to partially penetrate a superconductor breaking it up into many superconducting and normal regions.
Type II superconductors have been developed that can carry high currents as well as sustain very large magnetic fields.
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Some commonly used materials for both civilian and military applications are niobium-tin and niobium-titanium. Niobium-titanium is often chosen because of its superior mechanical properties. It has a critical magnetic field of 15 Tesla and a critical temperature of 10 K.
Among the many applications:
Among the many applications:

Magnetic Resonance Imaging
Japanese superconducting magnetically levitated train
Superconducting magnet used to detonate mines
Many other military and civilian applications are either contemplated or in use:
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High energy accelerators and detectors
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High energy accelerators and detectors

Magnetic energy power storage
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- Magnetic energy power storage
- Toroidal fusion reactors
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Superconductor Electronics

Josephson Junctions
Superconductor Electronics

Josephson Junctions

Superconductors

1 2

barrier

① ②

superconductors
Superconducting Quantum Interference Devices (SQUIDS)
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Very sensitive measurement of magnetic fields
Numerous civilian and military applications
Numerous civilian and military applications

Submarine detection
Numerous civilian and military applications

Submarine detection

Oil prospecting
Numerous civilian and military applications

Submarine detection

Oil prospecting

Precision sensors in scientific experiments
Numerous civilian and military applications

- Submarine detection
- Oil prospecting
- Precision sensors in scientific experiments
- Gravity waves
Numerous civilian and military applications

Submarine detection

Oil prospecting

Precision sensors in scientific experiments

Gravity waves

Tests of General Relativity
Numerous civilian and military applications

Submarine detection

Oil prospecting

Precision sensors in scientific experiments

Gravity waves

Tests of General Relativity

Medical diagnostics
SQUID Superconducting Quantum Interference Device in a medical application
SQUID Superconducting Quantum Interference Device in a medical application
Quantum Computing
Quantum Computing

Single pair box
Quantum Computing

Single pair box

barrier

superconducting electrodes
The single pair box utilizes a Josephson junction between two superconducting electrodes. Pairs can tunnel coherently through such junctions. This may serve as a qubit.
High Tc Superconductors
High Tc Superconductors

These materials can be superconducting at liquid nitrogen temperatures making possible long distance transmission of electric power as well as all of the other applications we have discussed but at higher temperatures.
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Nuclear electric power plants in remote areas.
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Nuclear electric power plants in remote areas.

Hydrogen economy.
Some of these applications we could foresee but most we could not.
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In doing fundamental science, perhaps, most important is what we cannot foresee.
Almost all of the technology we rely on for civilian and military use:
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Communications
Almost all of the technology we rely on for civilian and military use:

- Communications
- Computers
Almost all of the technology we rely on for civilian and military use:

- Communications
- Computers
- Electronics
Almost all of the technology we rely on for civilian and military use:

- Communications
- Computers
- Electronics
- Medical imaging
Almost all of the technology we rely on for civilian and military use:

- Communications
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- Electronics
- Medical imaging
- Laser surgery . . .
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- Einstein
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- Schrödinger
- Heisenberg
- Dirac

and many others.
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But their link to fundamental science is less well understood.
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In addition....
It is almost impossible to predict what technologies will flow from fundamental science.
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Maxwell, Lorentz, Einstein
(Electromagnetic Theory)
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Kammerling Onnes
(Superconductivity)
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Kammerling Onnes
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Mine detonator
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Schrödinger, Heisenberg
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Transistors, Computers
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Townes
(Millimeter Radiation)
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Laser Surgery
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Bloch, Purcell
(Solid-State Research)
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MRI
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In 1887 Edward Bellamy wrote, with a certain optimism:

If we could have devised an arrangement for providing everybody with music in their homes, perfect in quality, unlimited in quantity, suited to every mood, and beginning and ceasing at will, we should have considered the limit of human felicity already attained, and ceased to strive for further improvements.
Theory of Superconductivity

J. Bardeen, L. N. Cooper,† and J. R. Schrieffer‡

Department of Physics, University of Illinois, Urbana, Illinois

(Received July 2, 1957)

A theory of superconductivity is presented, based on the fact that the exchange between electrons resulting from virtual processes is attractive when the energy difference between the electron states involved is less than the phonon energy. It is favorable to form a superconducting phase when this attractive interaction dominates the repulsive screened Coulomb interaction. The normal phase is described by the Bloch individual-particle model. The ground state of a superconductor, obtained from a linear combination of normal state configurations in which electrons are virtually excited in pairs of opposite spin and momentum, is lower in energy than the normal state by an amount proportional to the average \( \langle \omega \rangle \), consistent with the isotopic effect. A mutually orthogonal set of excited states in one-to-one correspondence with those of the normal phase is obtained by specifying occupation of certain Bloch bands by the rest to form a linear combination of virtual pair configurations. The theory yields a second-order phase transition and a Meissner effect in the form suggested by Pippard. Calculated values of specific heats and penetration depths and their temperature variation are in good agreement with experiment. There is an energy gap for individual-particle excitations which decreases from about 3.5\( \Delta \) at \( T=0 \)K to zero at \( T_c \). Tables of matrix elements of single-particle operators between the excited-state superconducting wave functions, useful for perturbation expansions and calculations of transition probabilities, are given.

I. INTRODUCTION

The main facts which a theory of superconductivity must explain are (1) a second-order phase transition at the critical temperature, \( T_c \), (2) an electronic specific heat varying as \( \exp(-T/T_c) \) near \( T=0 \)K and other evidence for an energy gap for individual-particle-like excitations, (3) the Meissner-Ochsenfeld effect (\( B=0 \)), (4) effects associated with infinite conductivity (\( \sigma=0 \)), and (5) the dependence of \( T_c \) on isotopic mass, \( M \approx M \)-const. We present here a theory which accounts for all of these, and in addition gives good quantitative agreement for specific heats and penetration depths and their variation with temperature when evaluated from experimentally determined parameters of the theory.

When superconductivity was discovered by Onnes\(^1\) (1911), and for many years afterwards, it was thought to consist simply of a vanishing of all electrical resistance below the transition temperature. A major advance was the discovery of the Meissner effect\(^2\) (1933), which showed that a superconductor is a perfect diamagnet; magnetic flux is excluded from all but a thin penetration region near the surface. Not very long afterwards (1935), London and London\(^3\) proposed a phenomenological theory of the electromagnetic properties in which the diamagnetic aspects were assumed basic. F. London\(^4\) suggested a quantum-theretic approach to a theory in which it was assumed that there is somehow a coherence or rigidity in the superconducting state such that the wave functions are not modified very much when a magnetic field is applied. The concept of coherence has been emphasized by Pippard\(^5\), who, on the basis of experiments on penetration phenomena, proposed a nonlocal modification of the London equations in which a coherence distance, \( \xi_c \), is introduced. One of the authors\(^6\) pointed out that an energy-gap model would most likely lead to the Pippard version, and we have found this to be true of the present theory. Our theory of the diamagnetic aspects thus follows along the general lines suggested by London and by Pippard.

The Sommerfeld-Bloch individual-particle model (1928) gives a fairly good description of normal metals, but fails to account for superconductivity. In this theory, it is assumed that in first approximation one may neglect correlations between the positions of the electrons and assume that each electron moves independently in some sort of self-consistent field determined by the other conduction electrons and the ions. Wave functions of the metal as a whole are designated by occupation of Bloch individual-particle states of energy \( \varepsilon(k) \) defined by wave vector \( k \) and spin \( \gamma \), in the ground state all levels with energies below the Fermi energy, \( \delta \varepsilon \), are occupied; those above are unoccupied. Left out of the Bloch model are correlations between electrons brought about by Coulomb forces and interactions between electrons and lattice vibrations (or phonons).

\(^{†}\) This work was supported in part by the Office of Ordnance Research, U. S. Army. One of the authors (J. R. Schrieffer) was aided by a Fellowship from the Cuming Glass Works Foundation. Parts of the paper are based on a thesis submitted by Dr. Schrieffer in partial fulfillment of the requirements for a Ph.D. degree in Physics, University of Illinois, 1957.‡ Present address: Department of Physics and Astronomy, The Ohio State University, Columbus, Ohio.\(^1\) Present address: Department of Theoretical Physics, University of Birmingham, Birmingham, England.

\(^1\) W. K. Finn, Comm. Phys. Lab. Univ. Leiden, Nos. 119, 120, 121 (1911).


* This work was supported in part by the Office of Ordnance Research, U. S. Army.