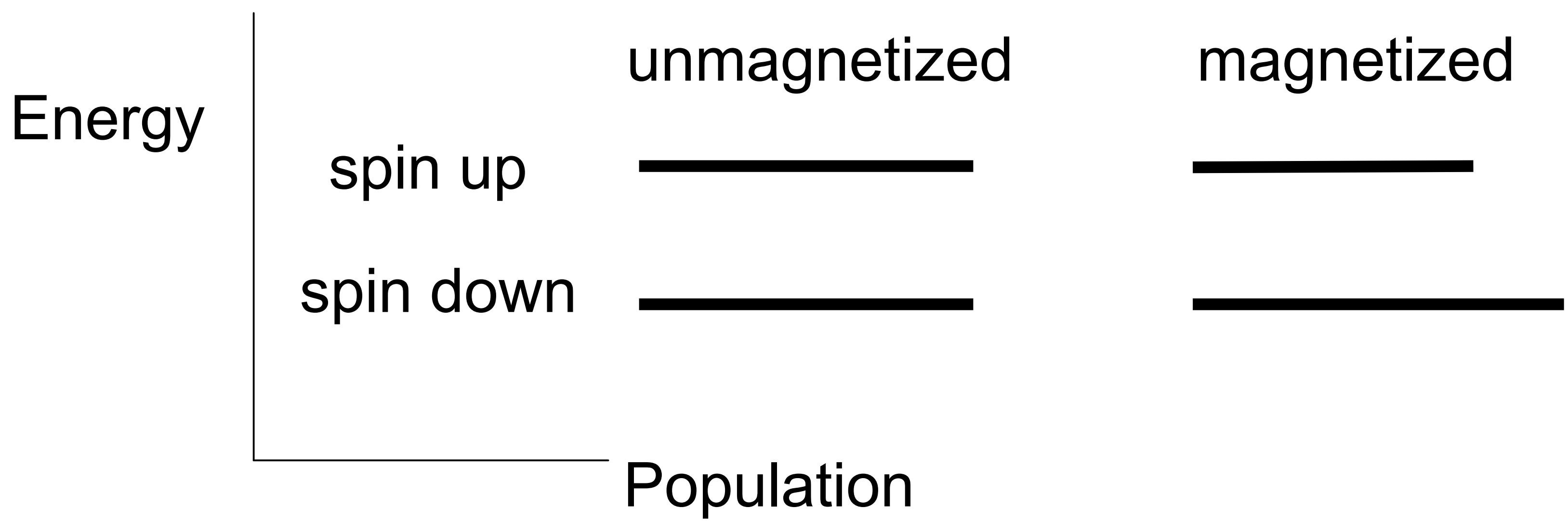
NMR and the BCS Theory of Superconductivity

Our NMR activities in the early 1950s (Norberg, Holcomb, Carver, Schumacher)

Overhauser dynamic nuclear spin polarization
Conduction electron spin susceptibility (Pauli)

Measuring the nuclear spin-lattice relaxation time, T_1, of the alkali metals: The time for the nuclear magnetization of an unmagnetized sample to be established



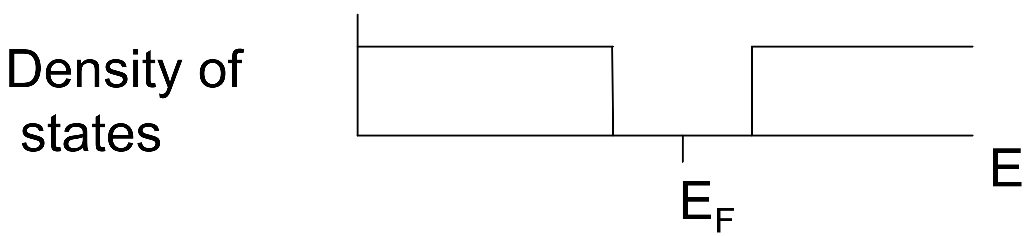
Superconductivity: some milestones

Discovery of the isotope effect (T_C~$M^{-1/2}$) by Maxwell and by Reynolds, Serin, Wright, and Nesbitt (1950)

Ginzburg-Landau theory of superconductivity(1950) (Ginzburg awarded Nobel Prize in 2003)

Pippard shows that the ***superconducting wave function extends over long distance*** (10^{-4} cm) (1950)

Evidence develops for ***a gap in the density of states*** in energy(1954)



An energy gap in the density of states?

Evidence for a gap in the density of states in superconductors (Corak,Goodman, Satterthwaite, and Wexler,1954)

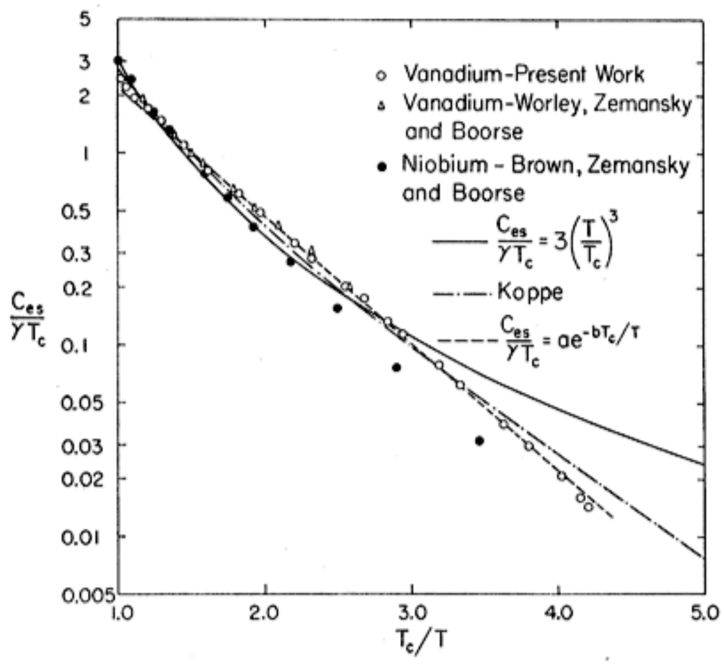


FIG. 1. Temperature dependence of the electronic specific heat of a superconductor.

Theory of the Meissner Effect in Superconductors

J. BARDEEN

University of Illinois, Urbana, Illinois

(Received January 3, 1955)

THE general features of the superconducting state are now well-established, although a good mathematical or detailed physical description is lacking. Pippard[1] has shown that the wave functions (range-of-order) of the electrons in the superconducting state extend over relatively large distances ($\sim 10^{-4}$ cm) and that the penetration depth does not vary much with magnetic field. The latter implies that a linear theory, in which only first-order changes of wave functions produced by the magnetic field are included, should be satisfactory. As pointed out particularly by Slater,[2] wave functions extending over large areas are favorable for a large diamagnetism. While it is thought that the Meissner effect ($\mathbf{B}=0$) follows rather generally from these considerations, it has been difficult to treat a specific model. One model, which is a modification of a degenerate free-electron gas, is discussed below.

Bardeen's colloquium in 1954

Bardeen says that ***a gap in the density of states at the Fermi energy, will probably explain the Meissner effect*** and thus lead to an explanation of superconductivity.

Bardeen tried to explain ***the origin of the gap using the electron-lattice interaction as suggested by the isotope effect,*** but did not have a complete theory.

CPS realized that ***nuclear relaxation in metals arose via the electrons close to the Fermi energy, so a gap should produce be a big change in T_1 in the superconducting state.***

We should ***measure T_1 in a superconductor*** !!

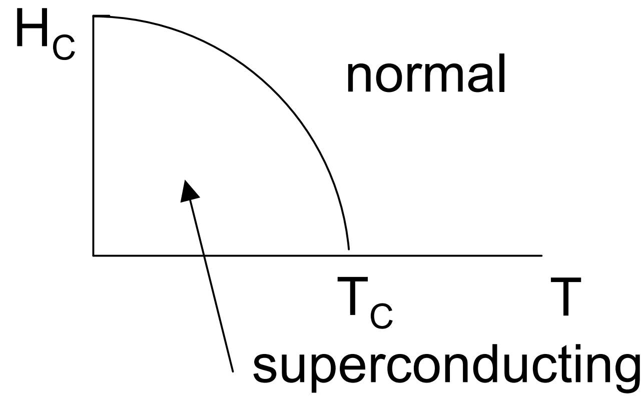
The crucial problem

Conventional NMR was done in large iron electromagnets or permanent magnets generating magnetic fields~7-10 kilogauss, with alternating magnetic fields operating at frequency~10MHz.

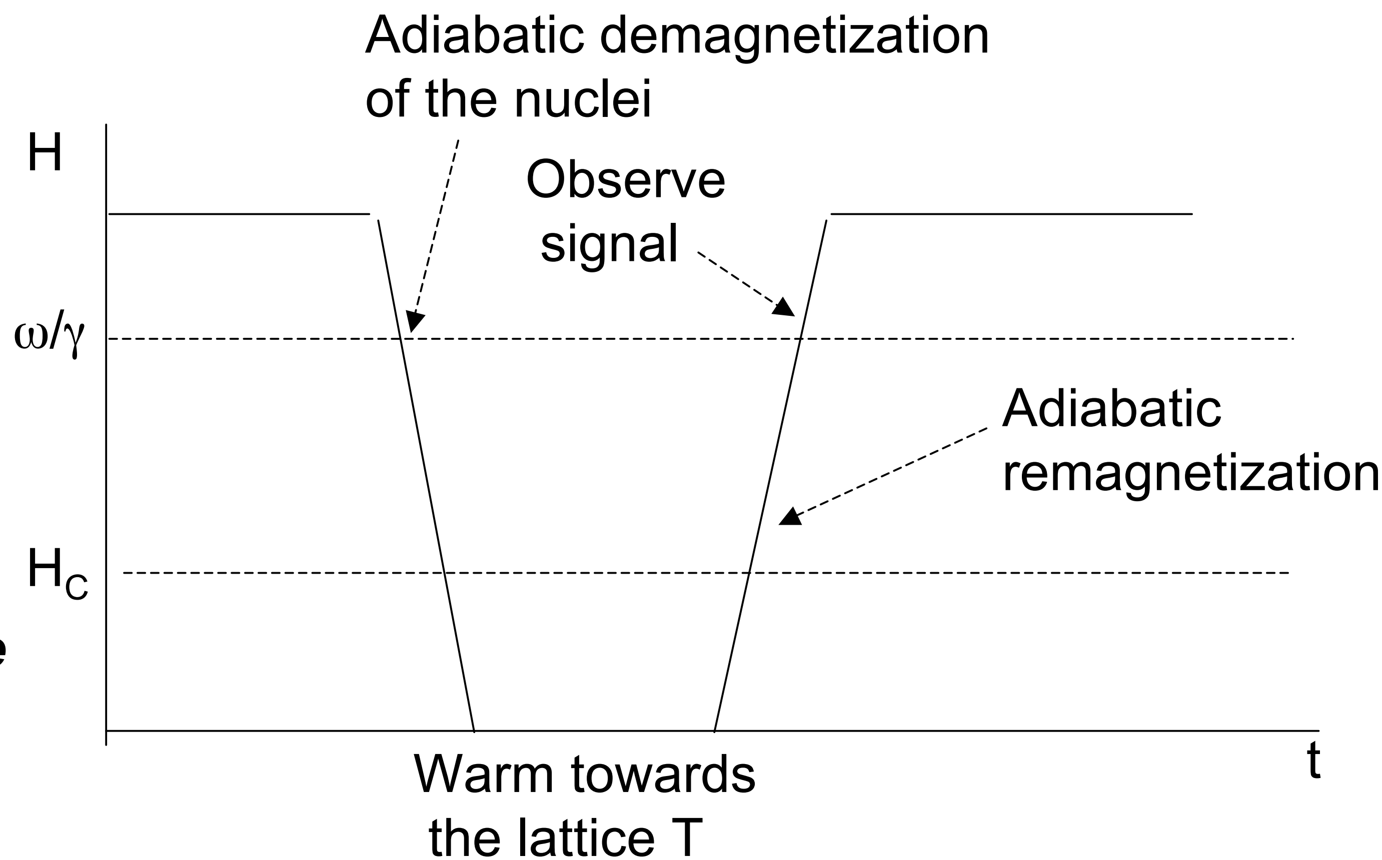
As a result of the Meissner effect (superconductors are perfect diamagnets) ,superconductors exclude the magnetic fields. **How can one do magnetic resonance in a superconductor?**

A solution to the experimental dilemma: Field Cycling

A strong magnetic field suppresses superconductivity.



Cycle between the normal and superconducting states,relaxing in the superconducting states, observing in the normal state



Which superconductor to study?

One must be able to cool below the superconducting transition.

One must be able to turn the magnetic field off and on in a time short compared to the nuclear spin-lattice relaxation time.

	^{207}Pb	^{199}Hg	^{115}In	^{27}Al
T_C(K)	7.19	4.13	3.40	1.17
abundance	21%	16%	95%	100%
ν(MHz) at 1 T	8.9	7.6	9.3	11.1
H_C (T=0)in gauss	803	412	293	105
T_1 at 1 K (msec)	$\approx$ 10 Too fast	$\approx$ 10 Too fast	$\approx$ 10 Too fast	**450**

^{27}Al is good except for the low T_C

Chuck Hebel (1952)

The properties of our apparatus
Cooling

Two choices to cool below 1.17 K:
adiabatic demagnetization
or
pump on ^{4}He (Rollin film)

This was our first experiment at such low temperatures so we chose pumping

Three Dewar system
(nitrogen,***outer He***, ***inner He***)We could reach 0.94K ($T=0.8T_C$) by pumping

The properties of our apparatus
The magnet

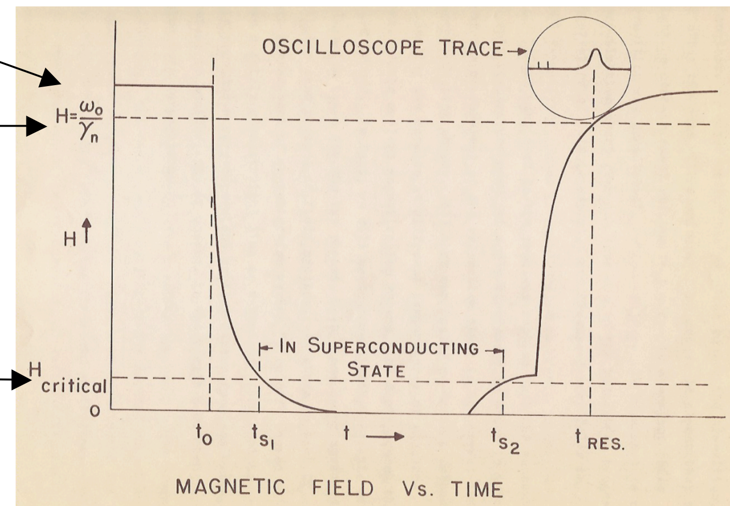
Conventional NMR was done at about 10,000 gauss in large iron magnets.

To achieve rapid switching, we made a special magnet from leftover Betatron iron sheets and worked at low fields of a few hundred gauss.

500 gauss

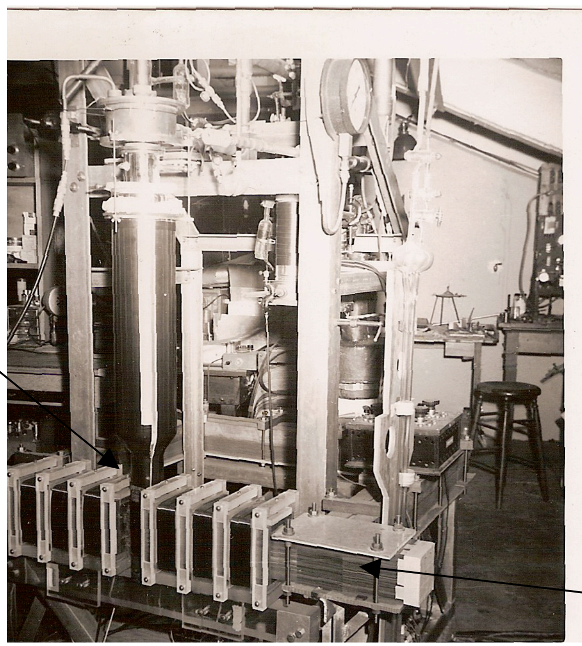
360 gauss
(400kHz)

105 gauss



1 msec.

Dewar
vessels



Laminated iron electromagnet

Conventional theory of T_1 mechanism in a metal

the electron-nuclear hyperfine coupling scatters the electron, flipping the electron spin up (down) and the nuclear spin down (up)

$$1/T_1 = C\int \underbrace{\left|\langle i|V_{en}|f\rangle\right|^2 \rho(E_f)}_{\text{Fermi's Golden Rule}}\underbrace{[1-f(E_f)]}_{\text{Final state Is empty}}\underbrace{f(E_i)\rho_i}_{\text{Sum over occupied initial states}} dE_i$$

$$\left|E_f - E_i\right| = \gamma_n \hbar H_o$$

$$V_{en} = \frac{8\pi}{3}\gamma_e\gamma_n\hbar^2\delta(r) I * S$$

$$I * S = I_z S_z + \frac{I^+S^- + I^-S^+}{2}$$

C depends on the nuclear spin Hamiltonian and thus is field dependent.

What did we expect? ***T_1 should be slower!***

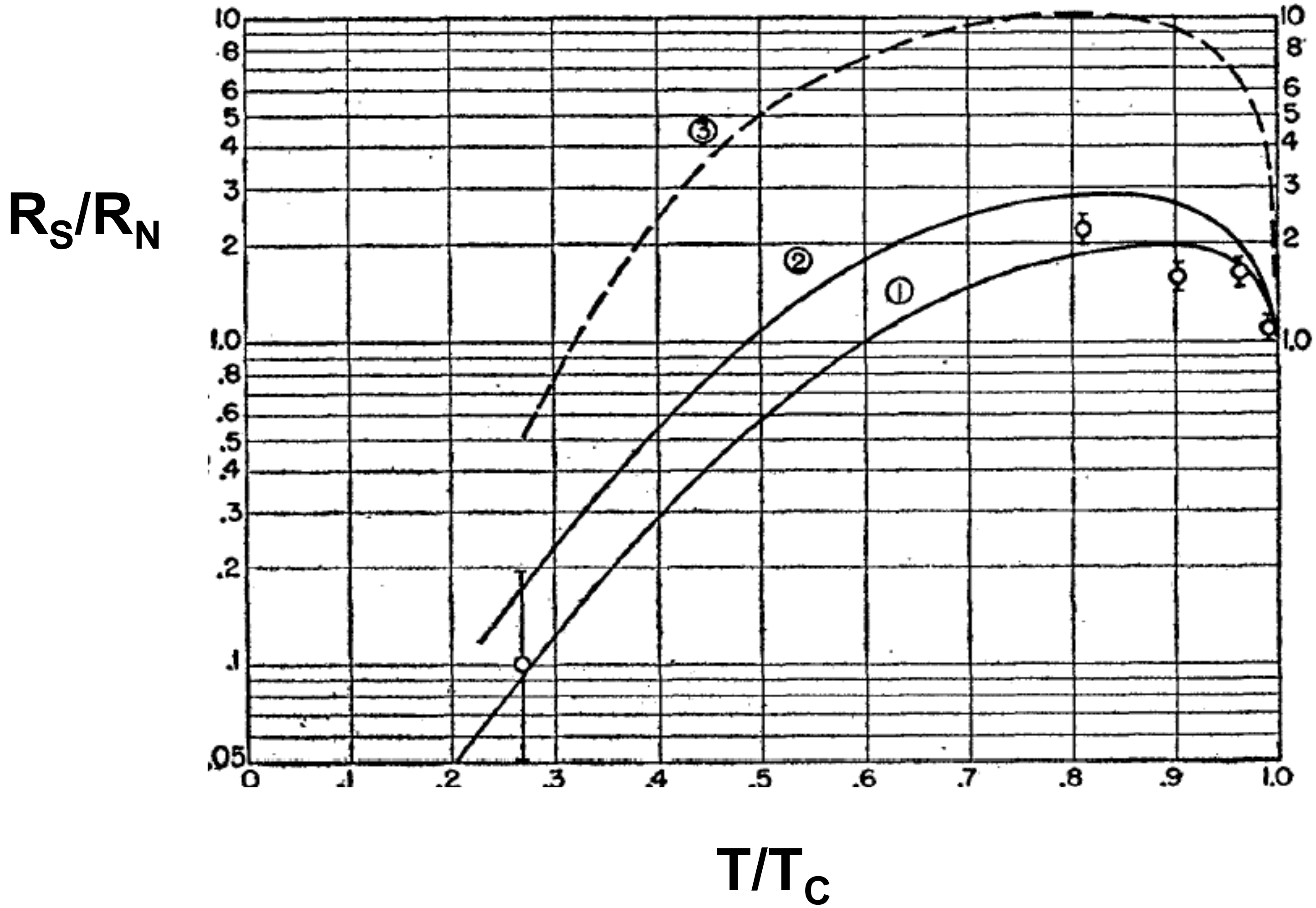
Two models:

- two fluid models: since only the "normal fluid" should be able to scatter ,and since the normal fluid density drops with T, T_1 should slow as T drops.

- A gap should inhibit excitations and thus T_1 should be slower below T_C

Experimental results (fall of 1956) ($R \equiv 1/T_1$)

T_1 is faster in the superconductor !



Direct observation of the energy gap (1956)

Transmission of Superconducting Films at Millimeter-Microwave and Far Infrared Frequencies*

R. E. GLOVER, III,† AND M. TINKHAM

Department of Physics, University of California, Berkeley, California

(Received September 4, 1956)

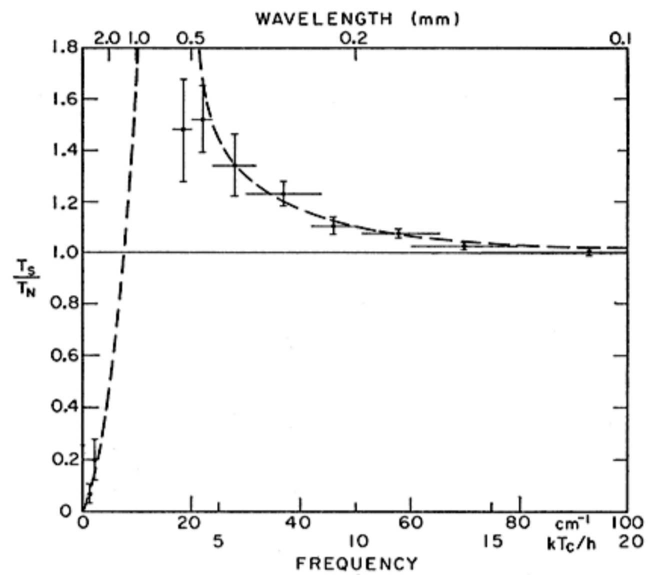
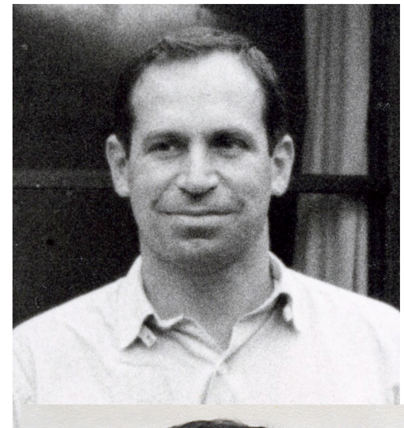


FIG. 1. Experimental transmission ratios of superconducting and normal states of a typical lead film (dc residual resistance 117 ohms; transmission in normal state $=\frac{1}{4}$) at $T/T_c=0.67\pm0.03$.

Meanwhile, some other developments:

Theory of the ***effect of lattice vibrations on the electron-electron coupling***
(J. Bardeen and D. Pines,
Phys. Rev **99**,1140 (1955).



Theory of the interaction of ***pairs of electrons above a filled Fermi sea,“***Cooper pairs”
(L.N. Cooper, Phys. Rev **104**,1189 (1956).

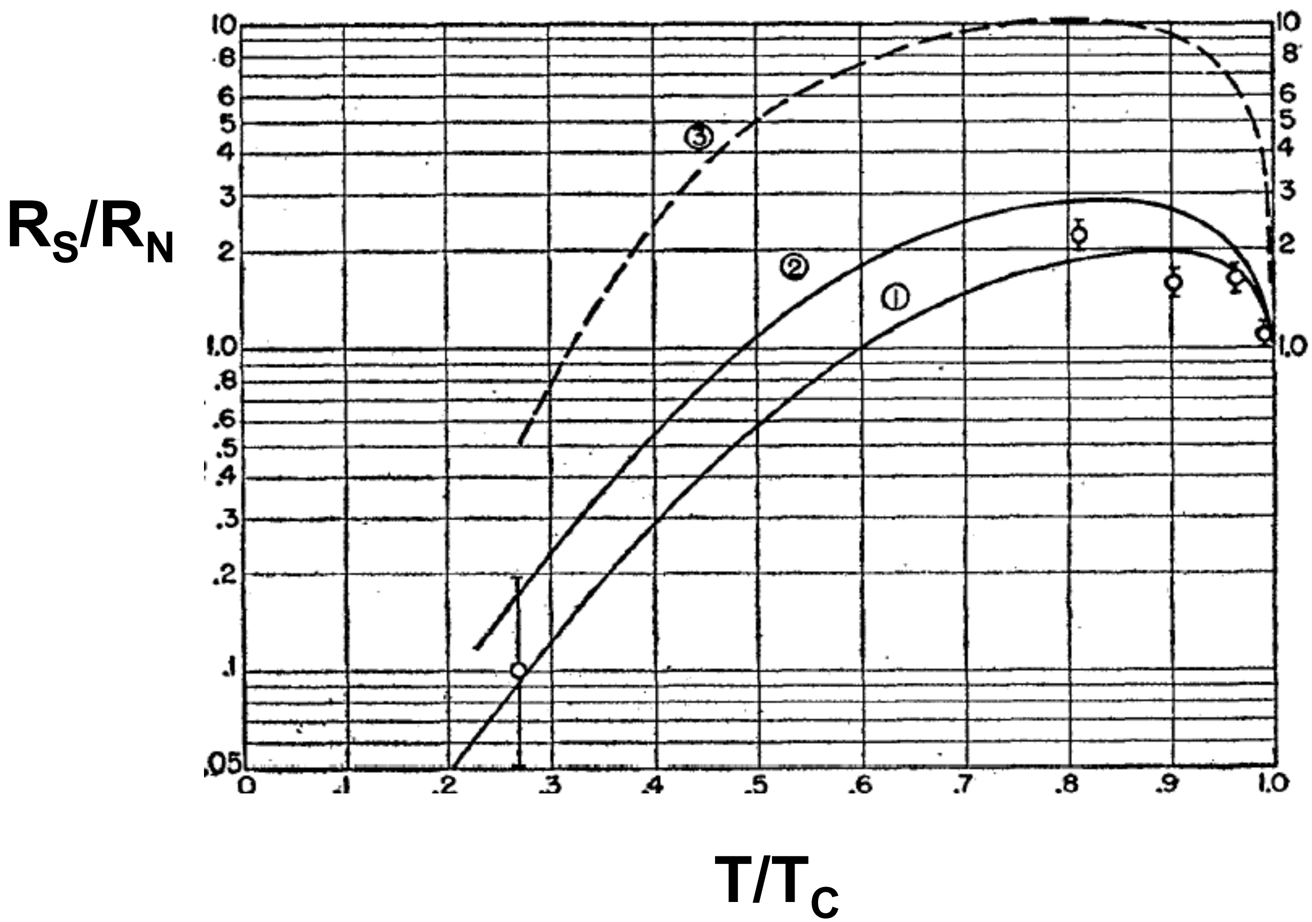
Announcement that ***Bardeen, Brattain, and Shockley*** had won the ***Nobel Prize*** for Physics for invention of the transistor
(October, 1956).

John Bardeen in his office in Urbana

Bob Schrieffer (1954)

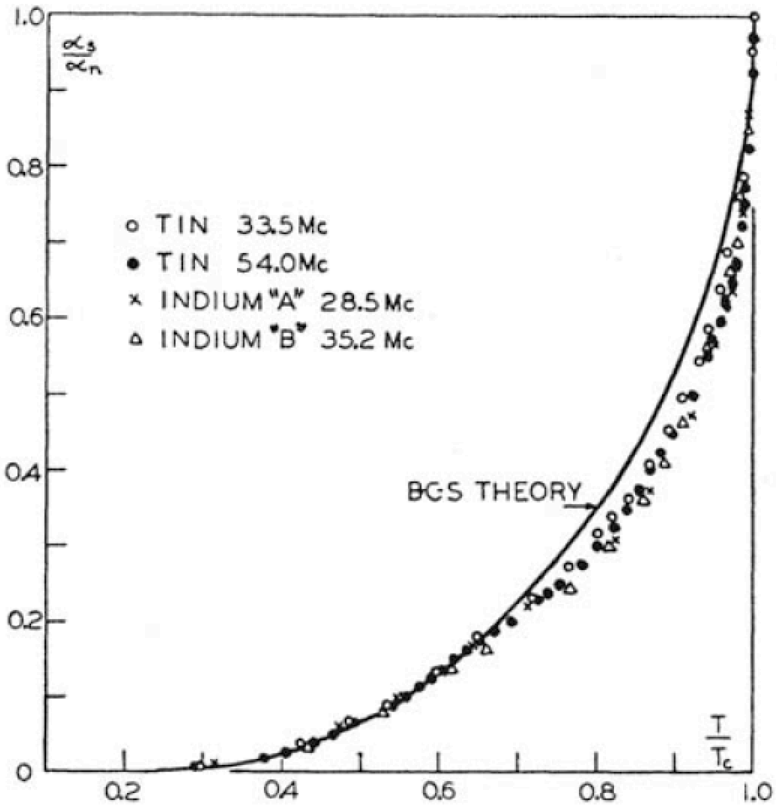
Spring 1957:Our calculations using the BCS theory ($R \equiv 1/T_1$)

Phys Rev **102**,901 (1957) and **113**,1504 (1959)



Three ways to deal with the Infinite density of states at the gap edge

Sound Absorption, Morse and Bohm (1957)



The precipitous drop in rate contrasts with the NMR rise. The two rates should have the same temperature dependence in a one electron theory

Since $E_i \cong E_f$, $1/T_1 = C\int |V|^2 \rho^2 \; f(1-f)dE$

Winter of 1957
Reconciling the NMR and sound absorption data

$$1/T_1 = C\int \underbrace{\left|\langle i|V_{en}|f\rangle\right|^2 \rho(E_f)}_{\text{Fermi's Golden Rule}} \underbrace{[1-f(E_f)]}_{\text{Final state Is empty}} \underbrace{f(E_i)\rho_i dE_i}_{\text{Sum over occupied initial states}}$$

The peaking of the BCS density of states explains the rise in NMR rate just below T_C.

For NMR we found that $\left|\langle i|V_{en}|f\rangle\right|^2 = V_{if}^2\left[V_1^2 + V_2^2\right]/2$,

$$E^2 = \varepsilon^2 + \Delta^2, \quad V_1^2 = 1 + \frac{\varepsilon\varepsilon'}{EE'}, \; V_2^2 = \frac{\Delta^2}{EE'} .$$

For sound absorption, BCS found that one needed

$$\left|\langle i|V_{en}|f\rangle\right|^2 = V_{if}^2\left[V_1^2 - V_2^2\right]/2 ,$$

The distinction arose from the ***pair nature of the wave function*** *and thus would* ***not*** *be present* ***in a one electron theory****!* ***The combined data therefore confirm the pair nature of the wave function.***

1958

A break-through in cooling(^{3}He price drops tenfold!)

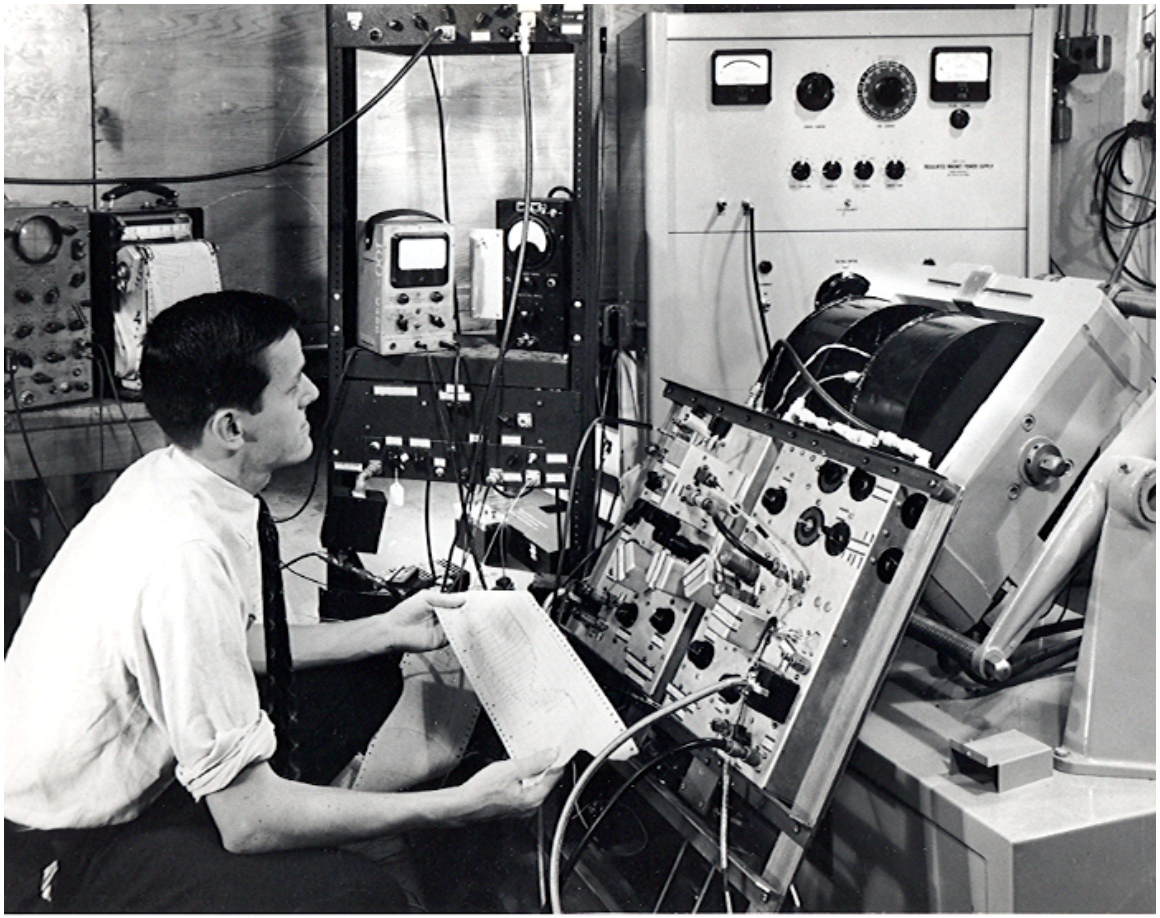
He^3 Cryostat for Measuring Specific Heat*

G. Seidel and P. H. Keesom
Purdue University, Lafayette, Indiana

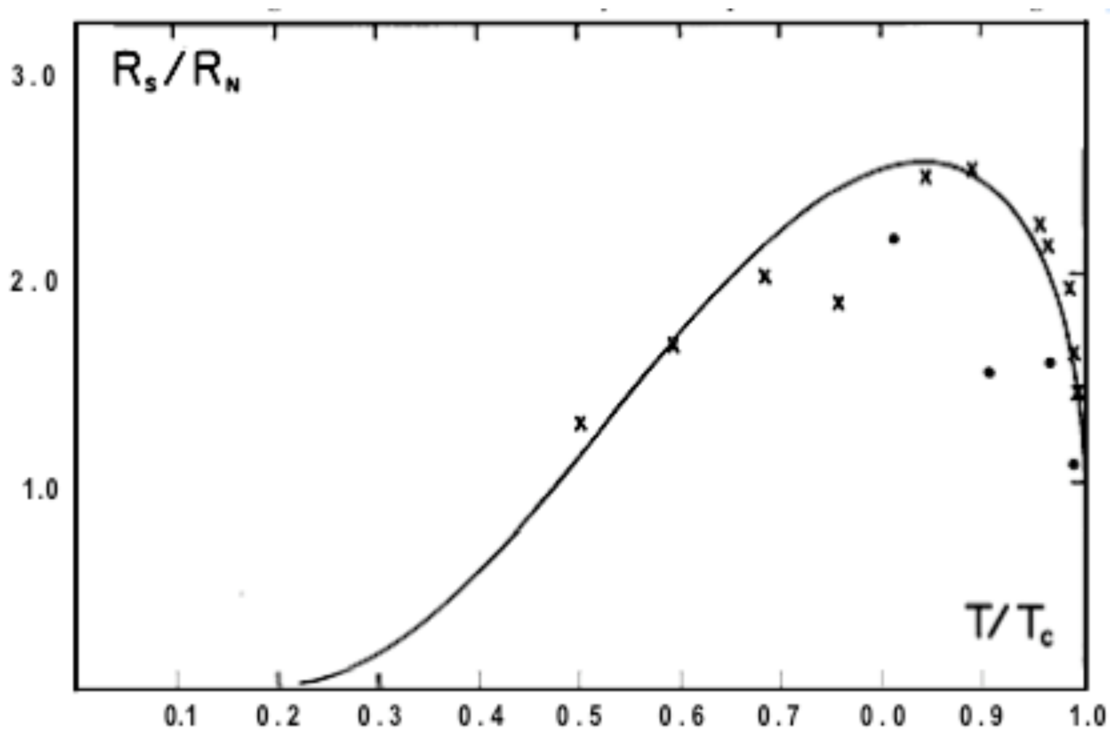
(Received April 25, 1958)

An apparatus employing a bath of liquid He^3 as a coolant has been developed for measuring specific heats below 1°K. Temperatures as low as 0.30°K can be maintained for long periods of time. A description of a simple thermal switch is also included.

Al Redfield, when a post doc with Nico Bloembergen
1953-1955



The combined data shown in Cooper's Nobel prize lecture



Solid dots:Hebel-Slichter **Crosses: Redfield-Anderson**

The solid line is the prediction using the BCS theory with level broadening

Conclusion

The BCS theory gives a complete explanation of the nuclear relaxation time in a superconductor.

The contrasting behavior of NMR relaxation rate with ultrasound absorption is explained in a natural manner by the pair nature of BCS wave function, providing proof of this central concept of the BCS theory.