BCS from 1952-57:
A Personal History

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BCS from 1952-57: A Personal History

- Before the Creation: Two steps on the road to BCS--the polaron wave function and the effective electron interaction that leads to superconductivity
- The Impact of BCS: I. Princeton: from early February to May, 1957: Explaining the Matthias “rules” for where superconductors live
- The Impact of BCS: II. Copenhagen: Summer, 1957: Nuclear Superconductivity
- John Bardeen’s experiment-based approach to theoretical problems--a paradigm for BCS and high T_c

Polarons: From 1952 in Urbana to 1957 in NYC

The motion of slow electrons in polar crystals is substantially modified by their strong coupling to the crystalline optical vibrations. The dimensionless coupling constant that characterizes that interaction is typically from ~ 3 to 6, so perturbation theory fails, and non-perturbative methods are required to understand the behavior of the polaron—the electron plus its co-moving cloud of phonons that increases its effective mass.

Bardeen to DP: Can obtaining a better description of such strong coupling provide insight into the role that electron-phonon coupling plays in bringing about superconductivity? See what you can learn.

DP to T.D Lee: Can we adapt Tomonaga's variational approach to the "intermediate coupling" problem of a nucleon coupled to mesons to the polaron problem of an electron coupled to optical phonons? Yes, we can and so a obtain reasonable ground state energy and polaron effective mass

Francis Low, T.D. Lee, and DP: discovered a simple and elegant expression for the polaron wavefunction, that resonated with Bob Schrieffer in the NYC subway four years later

PHYSICAL REVIEW VOLUME 88, NUMBER 4 NOVEMBER 1, 1952

The Motion of Slow Electrons in Polar Crystals

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(Received September 17, 1952)

WE have investigated the low-lying energy levels of a conduction electron in a polar crystal, using the "intermediate coupling" variational technique introduced by Tomonaga.[1] This problem is of considerable interest because of the strong interaction between the electron and the ionic polarization it produces in its motion through the crystal. The electron may be pictured as accompanied by a cloud of phonons (i.e., the associated waves of ionic polarization), and the combined system (electron plus associated phonon cloud) is known as a polaron. The strength of the electron-lattice interaction furnishes a measure of the average number of phonons in the cloud around the electron, and hence of the effective mass of the polaron.

PHYSICAL REVIEW VOLUME 90, NUMBER 2 APRIL 15, 1953

The Motion of Slow Electrons in a Polar Crystal

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(Received December 31, 1952)

We use a variational method of calculation, which we will show is equivalent to a simple canonical transformation. We choose for our trial wave function

$$\psi = U\psi_0, \tag{10}$$

where ψ_0 is the eigenstate of the unperturbed Hamiltonian with no phonons present, i.e., the "free" vacuum state. Specifically, ψ_0 is defined by

$$a_k\psi_0 = 0, \quad (\psi_0, \psi_0) = 1, \tag{11a}$$

and

$$U = \exp\{\textstyle\sum_k (a_k^* f(\mathbf{k}) - a_k f^*(\mathbf{k}))\}, \tag{11b}$$

where $f(\mathbf{k})$ will subsequently be chosen to minimize the energy. It is clear that U is a unitary operator, so that ψ is normalized. Furthermore, viewed as a unitary transformation, U is a displacement operator on a_k and a_k^*, since

$$U^{-1}a_k^*U = a_k^* + f^*(\mathbf{k}), \quad U^{-1}a_kU = a_k + f(\mathbf{k}). \tag{12}$$

Our variational calculation, which is based on the use of the state vector ψ [Eq. (10)] is closely related to the approximation introduced by Tomonaga in his treatment of the coupling between mesons and nucleons, which was subsequently named the "intermediate coupling" approximation.

A Collective Description of Electron Interaction in Metals: 1952-54

*In Urbana, I continued to work by correspondence with David Bohm (Sao Paolo) on writing up my Ph.D. thesis work that established plasmons as the dominant long wavelength excitation in metals (and most other solids) and subsequently made it possible to justify the widespread use of the independent electron model and calculate the corrections to its electronic properties coming from the screened Coulomb interaction

*I then started work on extending that description to the coupled system of electrons and phonons with the goal of explaining how, in the face of a much larger Coulomb interaction--L.D. Landau-"you cannot repeal Coulomb's law"--it was their coupling to phonons that determined the superconducting transition temperature

*The "aha" moment with John Bardeen that led to our derivation of the interaction responsible for superconductivity

PHYSICAL REVIEW VOLUME 92, NUMBER 3 NOVEMBER 1, 1953

A Collective Description of Electron Interactions: III. Coulomb Interactions in a Degenerate Electron Gas

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AND

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(Received May 21, 1953)

The behavior of the electrons in a dense electron gas is analyzed quantum-mechanically by a series of canonical transformations. The usual Hamiltonian corresponding to a system of individual electrons with Coulomb interactions is first re-expressed in such a way that the long-range part of the Coulomb interactions between the electrons is described in terms of collective fields, representing organized "plasma" oscillation of the system as a whole. The Hamiltonian then describes these collective fields plus a set of individual electrons which interact with the collective fields and with one another via short-range screened Coulomb interactions. There is, in addition, a set of subsidiary conditions on the system wave function which relate the field and particle variables. The field-particle interaction is eliminated to a high degree of approximation by a further canonical transformation to a new representation in which the Hamiltonian describes independent collective fields, with n' degrees of freedom, plus the system of electrons interacting via screened Coulomb forces with a range of the order of the inter electronic distance. The new subsidiary conditions act only on the electronic wave functions; they strongly inhibit long wavelength electronic density fluctuations and act to reduce the number of individual electronic degrees of freedom by n'. The general properties of this system are discussed, and the methods and results obtained are related to the classical density fluctuation approach and Tomonaga's one-dimensional treatment of the degenerate Fermi gas.

PHYSICAL REVIEW VOLUME 92, NUMBER 3 NOVEMBER 1, 1953

A Collective Description of Electron Interactions: IV. Electron Interaction in Metals

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(Received May 21, 1953)

The effects of the Coulomb interaction between free electrons in an electron gas are considered for a variety of phenomena. The analysis is based on the collective description, which describes the long-range correlations in electronic positions (due to the Coulomb force) in terms of the collective oscillations of the system as a whole. It is shown that an independent electron model should provide a good description of the electrons in a metal in many cases of interest. The ground state energy of the free electron gas is determined, and an estimate of the correlation energy is obtained, with results in good agreement with those of Wigner. The exchange energy is shown to be greatly reduced by the long-range correlations, so that its effect on the level density and the specific heat is comparatively slight, leading to an electronic specific heat for Na which is approximately 80 percent of the free-electron value. The possible ferromagnetism of a free-electron gas is investigated, and it is found that the long-range Coulomb correlations are such that a free-electron gas will never become ferromagnetic (no matter how low the density). The excitation of the collective oscillations by a fast charged particle is studied, and the semiclassical results obtained by Bohm and Pines are verified by a quantum-mechanical calculation. The results are applied to the experiments of Ruthermann and Lang on the scattering of electrons by thin metallic films and to experiments on the stopping power of light metals for fast charged particles, with resulting good agreement between theory and experiment.

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Electron-Phonon Interaction in Metals*

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(Received April 4, 1955)

The role of electron-electron interactions in determining the electron-phonon interaction in metals is investigated by extending the Bohm-Pines collective description to take into account the ionic motion. Collective coordinates are introduced to describe the long-range electron-ionic correlations, and it is shown by a series of canonical transformations that these give rise to plasma waves and to coupled electron-ion waves which correspond to longitudinal sound waves. The dispersion relation for the sound waves is identical with that derived by Toya and Nakajima by self-consistent field methods. The velocity of these sound waves is calculated from first principles for sodium and is found to be in good agreement with experiment. The effective matrix element for the electron-phonon interaction is determined and is found to be identical for long wavelengths with that found earlier by Bardeen using a self-consistent field method which neglects exchange and correlation effects. The agreement with the earlier work is explained by the fact that the residual electron-electron interaction is of quite short range, so that an independent-particle treatment is rather well justified. The effects of Coulomb correlations on superconductivity are likewise shown to be small, so that the neglect of Coulomb interactions in the formulation of the superconductivity problem is justified.

As a first step, we add a field-energy term to our initial Hamiltonian, (2.14) so that our Hamiltonian is now

$$H = H_1 + \tfrac{1}{2} \sum_{|\mathbf{k}|<k_c} P_k^* P_k. \tag{4.1}$$

The P_k are here a quite arbitrary set of field variables, as yet undefined, which commute with all operators appearing in H_1. In order that the energy and number of degrees of freedom of our system remain unchanged by this field, we must further impose a set of subsidiary conditions on our combined system wave function:

$$P_k \Psi = 0 \quad (|\mathbf{k}| < k_c). \tag{4.2}$$

We now wish to transform to a representation in which the P_k will describe independent plasma oscillations. In the absence of ionic motion, the ρ_k in the long-wave-length limit describe almost free collective oscillation, and the appropriate transformation would relate the P_k to the ρ_k. In our case, since the electrons also interact with the phonons, we would expect that it is only some linear combination of density fluctuations and phonon field variables which would carry out uncoupled collective plasma oscillation. Thus we are led to try a canonical transformation which relates the P_k to both ρ_k and the q_k. The first transformation we consider is generated by

$$S = \sum_{|\mathbf{k}|<k_c} (-iM_k \rho_{-k} + u_k q_{-k}) Q_k, \tag{4.3}$$

where Q_k is the coordinate conjugate to the plasma field momentum P_k, and u_k is a real constant to be determined. After this transformation our subsidiary condition, (4.2) becomes

$$e^{-iS/\hbar} P_k e^{iS/\hbar} \Psi = [P_k - iM_k \rho_{-k} + u_k q_{-k}] \Psi = 0. \tag{4.4}$$

Our Hamiltonian (4.1) is transformed to

$$\begin{aligned} H = {} & \sum_{\kappa} E_\kappa c_\kappa^* c_\kappa + \tfrac{1}{2} \sum_{|\mathbf{k}|<k_c} \{p_k^* p_k + (\Omega_k^2 - \omega_k^2) q_k^* q_k\} \\ & + \tfrac{1}{2} \sum_{|\mathbf{k}|<k_c} \{P_k^* P_k + (\omega_p^2 + u_k^2) Q_k^* Q_k\} \\ & + \sum_{|\mathbf{k}|<k_c} \{v_k^i - iM_k u_k\} q_k \rho_k + \sum_{|\mathbf{k}|<k_c} u_k \rho_k^* Q_k \\ & - \sum_{|\mathbf{k}|<k_c,\kappa} M_\kappa \frac{\hbar \mathbf{k}}{m} \cdot (\boldsymbol{\kappa} - \tfrac{1}{2}\mathbf{k}) c_\kappa^* c_{\kappa-k} Q_k \\ & + \tfrac{1}{2} \sum_{|\mathbf{k}|>k_c} \{p_k^* p_k + \Omega_k^2 q_k^* q_k\} + \sum_{|\mathbf{k}|>k_c} v_k^i q_k \rho_{-k} \\ & + \tfrac{1}{2} \sum_{|\mathbf{k}|>k_c} M_k^2 \rho_{-k} \rho_k. \end{aligned} \tag{4.5}$$

John Bardeen and David Pines, "Electron-Phonon Interaction in Metals," *Phys. Rev.* **99**, 1140 (1955).

Except for terms which correspond to transitions which nearly satisfy the conservation of energy,

$$|E_{\kappa'}-E_{\kappa}\pm\hbar\omega_k|<\Delta E, \tag{5.1}$$

the electron-lattice interaction can be eliminated by a canonical transformation such that in the final Hamiltonian the lattice oscillators are not coupled with the electrons. It is the remaining interaction terms which do satisfy (5.1) that are responsible for scattering of electrons and also presumably account for superconductivity. The value of ΔE always can be chosen sufficiently small so that these terms have a negligible effect on the electron-lattice matrix element and on the vibrational frequency. On the other hand, they cannot be treated by perturbation theory, and so can have a pronounced effect on the electron wave functions.

In the theory of superconductivity, then, one need only consider virtual transitions which satisfy an expression of the form (5.1). This approach was followed by one of the authors,[12] with ΔE chosen to be of the order of the electron-lattice interaction energy resulting from these virtual transitions. This gives [see Eq. (3.35)]

$$\Delta E\sim N(E)\langle|v_kq_k|^2\rangle_{\mathrm{Av}}, \tag{5.2}$$

which is an order of magnitude or so larger than kT_c (T_c= transition temperature) for most superconductors. If one assumes that these interactions contribute to the superconducting state, but not to the normal state, one would have far too large an energy difference between the two phases. About the correct order of magnitude for this energy difference is obtained if we arbitrarily take $\Delta E\sim kT_c$; that is if electrons with energies within $\sim kT_c$ of the Fermi surface have their energies lowered by $\sim kT_c$. Undoubtedly the virtual transitions determined by (5.2) contribute to both the normal and superconducting states, and only a very small fraction of the interaction energy is involved in the change of state. It is evident that better pictures of both the normal and superconducting phases are required. The equations we have presented here should provide a good basis for development of an adequate theory.

The 1954 Solvay Congress

*I reported on my work with Bohm, my justification of the independent electron model of metals, and my work with Bardeen in the opening talk at the Solvay Congress on “The Electron Theory of Metals”

*In the audience were Lawrence Bragg, Christian Moeller, Neville Mott, Lars Onsager, **Wolfgang Pauli**, John Van Vleck, Herbert Frohlich, Charlie Kittel, Bernd Matthias, Pierre Aigrain, C.J. Gorter, Ilya Prigogine, Brian Pippard, Jacques Friedel

*I began by noting that our results showed that Heisenberg’s attempt to use the long range part of the Coulomb interaction to get superconductivity could not work, and that the interaction responsible for superconductivity was a combination of the direct screened Coulomb interaction and the screened electron-phonon induced interaction.

*Pauli’s reaction—during and after my talk

The immediate impact of BCS: from early Feb to May 1957, in Princeton

*News of the breakthrough arrived in early February in a letter from John enclosing a dittoed advance copy of their PRL

*Worked for 3 days with Elihu Abrahams and Philippe Nozières to flesh out the details

*Was then able to include BCS as part of my spring semester Princeton University graduate course on the quantum theory of solids, and to invite Bardeen for a colloquium on the details of BCS

*As part of my course, I explored whether the Bardeen-Pines interaction could explain the Matthias rules for the occurrence of superconductivity in the periodic system, and found that it could.

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Superconductivity in the Periodic System

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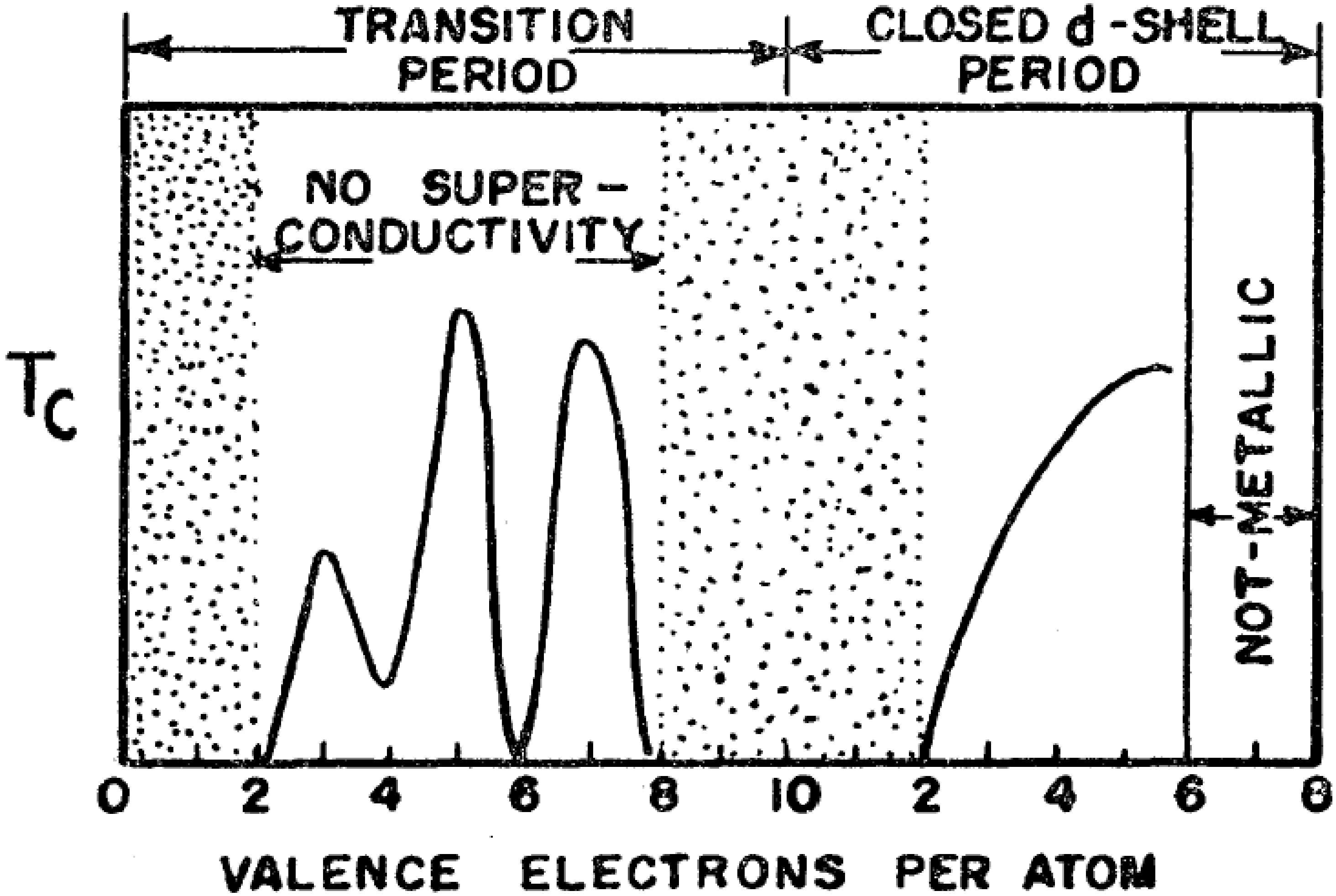
(Received May 27, 1957)

The empirical regularities in the appearance of superconductivity in the periodic system discussed by Matthias are considered in the light of the microscopic theory of superconductivity proposed by Bardeen, Cooper, and Schrieffer. A simple model of electrons and ions interacting via screened Coulomb forces is used to describe the electron-lattice interaction. With the aid of this model, it is shown how the theory of Bardeen, Cooper, and Schrieffer provides both a satisfactory criterion for the appearance of superconductivity and a good qualitative account of the variation in transition temperature from one metal to another.



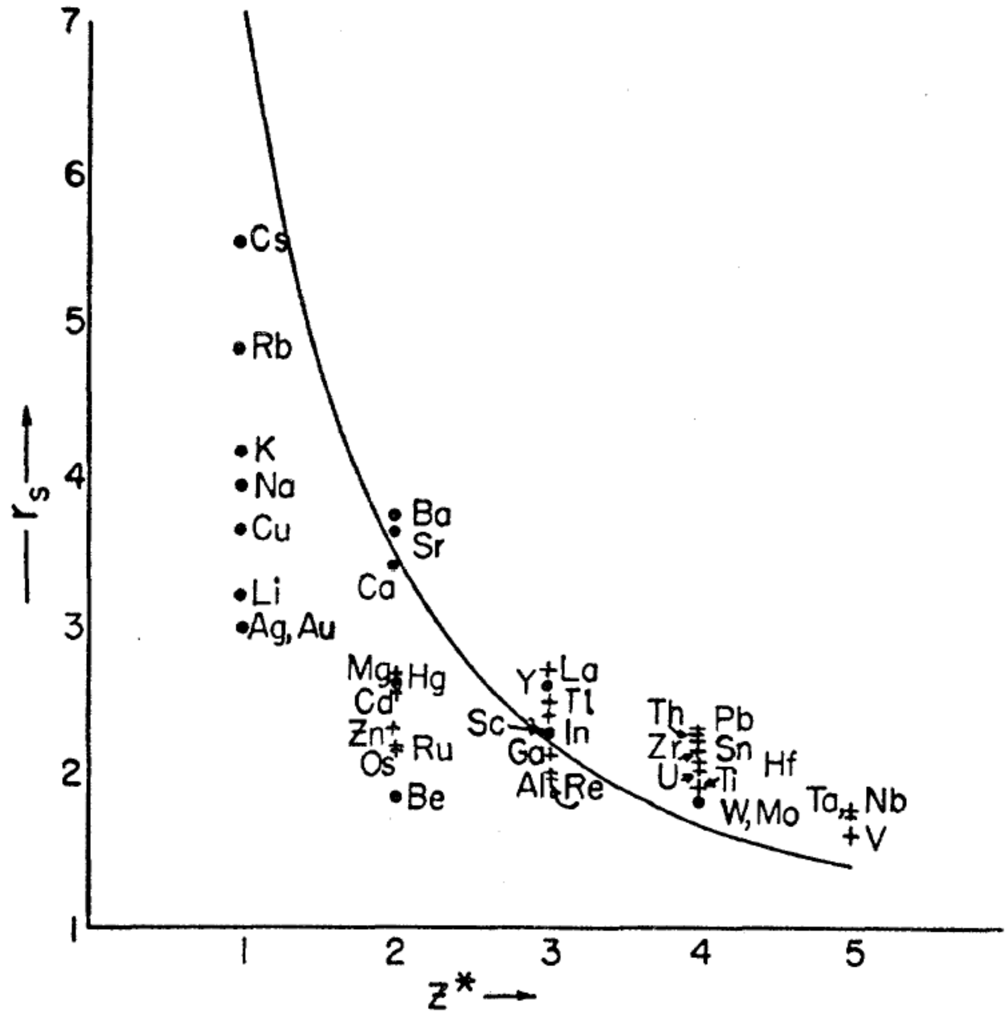
The behavior of T_c as a function of Z (after Matthias [B. Matthias, in *Progress in Low Temperature Physics*, ed. C.J. Gorter {North-Holland, Amsterdam, 1957}, Vol. 2]).

$$-V_N=\left\langle -\frac{4\pi N(Z^*)^2e^2}{M}\frac{4\pi e^2}{[(\mathbf{k}-\mathbf{k}')^2+k_s{}^2]^2}\frac{(\mathbf{k}-\mathbf{k}')^2}{s^2(\mathbf{k}-\mathbf{k}')^2}\right.$$

$$\left.+\frac{4\pi e^2}{(\mathbf{k}-\mathbf{k}')^2+k_s{}^2}\right\rangle_{\mathrm{Av}},$$

$$-V_U=\left\langle -\frac{4\pi N(Z^*)^2e^2}{M}\frac{4\pi e^2}{[(\mathbf{k}-\mathbf{k}')^2+k_s{}^2]^2}\frac{(\mathbf{k}-\mathbf{k}')^2}{s^2k_D{}^2}\right.$$

$$\left.+\frac{4\pi e^2}{(\mathbf{k}-\mathbf{k}')^2+k_s{}^2}\right\rangle_{\mathrm{Av}},$$



The critical r_s for superconductivity as a function of Z^*. The superconducting elements are denoted by +; the nonsuperconducting elements by •.

BCS in Copenhagen: Summer 1957

*Arrival at NBI in early June 1957,
for 3 months

*Invited to give lectures on BCS

*Warning about potential N. Bohr reaction

*DP first lecture

* Nuclear Superconductivity

SHORT CONTRIBUTION

NUCLEAR SUPERCONDUCTIVITY

BY

D. PINES †

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I would like, for a moment, to consider the many-body problem in the nucleus in the light of what we now know about an analogous many-body problem, that of electrons in metals. In ordinary metals, it is found experimentally that the low-lying excitations of the system possess an essentially individual particle character. The specific heat depends linearly on the temperature, and the Fermi surface is well-defined, as is evidenced by the de Haas-van Alphen effect. The main problem which faces the theoretical physicist is that of reconciling these facts with the rather strong long-range Coulomb interactions between the electrons. This may be done, however, by taking into account properly the polarization effects produced by the Coulomb interactions. When this is done, (by, for instance, introducing the plasma field to describe the polarization waves), one obtains a system of "effective" electrons, which interact through a screened Coulomb interaction with a range of the order of the inter-particle spacing. The "effective" electrons consist of the original electrons plus their associated screening clouds. It is then feasible to show that the elementary excitations of the "effective" electron system possess the desired individual particle character, although a completely satisfactory mathematical proof of this fact does not yet exist.

This situation is rather similar to that obtaining with the shell model for the nucleus, and one might hope for a similar happy ending here. I do not, however, believe that we will find it in just this form. To see why, let us consider the "unusual" metals, the superconductors.

Recently Bardeen, Cooper and Schrieffer [1]) have proposed a theory

†) National Science Foundation Senior Post-Doctoral Fellow, 1957-58; Permanent address: Princeton University, Princeton, N.J.

of superconductivity which is satisfactory in all major respects. They propose that the following criterion distinguishes between superconductors and ordinary metals: in superconductors the effective interaction between the electrons near the Fermi surface is attractive. Thus the phonon-induced electron interaction, which is attractive for electrons near the Fermi surface, must be more attractive than the screened Coulomb interaction is repulsive. This criterion may be shown to work quite well in practice.

They then consider the consequences of an effective averaged attractive interaction between electrons near the Fermi surface. They show that the wave functions of the system are markedly altered. They find for the ground state an essentially many-body wave function which corresponds to the coherent virtual excitation of all pairs of electrons of opposite spin and momentum near the Fermi surface. Further, they show it takes a finite amount of energy to excite any individual electron out of the coherent assembly, that is, the infinite system possesses an energy gap in its excitation spectrum which shows up in an exponentially varying specific heat at sufficiently low temperatures. Bardeen, Cooper and Schrieffer then go on to calculate a number of other superconducting properties, including the specific heat, Meissner effect, and phase transition, finding in each case good agreement with experiment.

What Bardeen, Cooper and Schrieffer have derived is, in fact, quite general, and should apply to any system of fermions for which the averaged interaction between the particles near the Fermi surface is attractive. In their theory, no matter how weak the attractive interaction may be, it will give rise to coherent many-body states for the particles near the Fermi surface and an energy gap in their excitation spectrum. I think that we will all agree that the effective interaction between nucleons in the nucleus is attractive. We might accordingly expect the BCS theory to apply to nuclei.

I should therefore like to suggest that nuclei resemble superconductors, in that the attractive interaction between the nucleons makes necessary the introduction of coherent many-body states and gives rise to an energy gap in the nuclear excitation spectrum. I believe that this effect must be taken into account in any proper treatment of the nuclear many-body problem. Bohr and Mottelson and I have been discussing these matters during the past summer in Copenhagen, and they will have more to say on this topic later in the meeting.

Reference

[1]) J. Bardeen, L. N. Cooper and J. R. Schrieffer, Phys. Rev. **106** (1957) 162

Discussion

A. Bohr: The important point from the nuclear point of view which is brought out by the work of Bardeen *et al.* is the major modification occurring at the Fermi surface of a Fermi gas in which there are attractive interactions. This modification extends over a certain energy region, representing a characteristic interaction energy. In the nuclear case it is quite possible that this interaction energy is small compared to the Fermi energy and therefore it should not effect, say, the calculation of the average binding energy of the particles in the nucleus. However, it is of course of great relevance for such problems as the justification of the shell model. Such a change of the Fermi surface would imply for instance that as we go to heavier and heavier nuclei there would finally be no shell structure because the distance between the shells would go to zero as the nucleus became heavier and it would finally be smaller than the region for which the whole spectrum was modified. Of course one should say that these effects are to some extent another way of talking about the configuration interactions which take place between neighbouring single particle levels and which have already been considered for some time, but this way of talking throws an interesting light on the problem. It is also possible—this remains to be seen—that one may be able to use some of the methods suggested in the electron problem to treat configuration interactions in the nucleus from a different point of view. In particular, the emphasis on correlations in which the particles remain in pairs is rather interesting. It is also somewhat suggestive that the gap in the energy spectrum which Bardeen and his collaborators obtained may have something to do with the gap existing in the intrinsic spectra of nuclei which is discussed in another session †.

Brueckner †† pointed out a difference in character between interactions in a superconductor and in a nucleus. In the superconductor the interaction between pairs was repulsive except for a very small region near the Fermi surface, whereas in a nucleus the interaction was attractive everywhere. There was no group of states in a nucleus where the interaction was of opposite sign from the bulk of the states. This might cause difficulty in carrying over the deductions of Bardeen to the nuclear case.

Eden asked how big the energy gap was likely to be in the case of a nucleus, in order to enable one to estimate the effect on calculations.

Mottelson: It is probably of the order of 1 MeV, and therefore not of first significance in calculating the total energy of the nucleus. But it will be very important for the discussion of the nuclear levels that lie within an energy interval of that order of magnitude.

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† B. R. Mottelson, Lecture in session on unified model and discussion following.

†† Editor's note: Brueckner has stated that on later examination these remarks seemed to be incorrect; however he thought they could remain in the proceedings.

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Possible Analogy between the Excitation Spectra of Nuclei and Those of the Superconducting Metallic State

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(Received January 7, 1958)

The evidence for an energy gap in the intrinsic excitation spectrum of nuclei is reviewed. A possible analogy between this effect and the energy gap observed in the electronic excitation of a superconducting metal is suggested.

It thus appears that there may exist interesting similarities between the low-energy spectra of nuclei and of the electrons in the superconducting metal. However, it must be stressed that the former are significantly influenced by the finite size of the nuclear system. Thus, the energy gap is observed to decrease with A, and the present data are insufficient to indicate the limiting value for the gap in a hypothetical infinitely large nucleus. Moreover, the degrees of freedom associated with the variation in shape play an especially important role in the low-energy nuclear spectra.

John Bardeen’s Bottom-up, Experiment- based Approach to Theoretical Physics: BCS as a worked example

*Focus first on the experimental results via reading and personal contact

*Develop a phenomenological description that ties different experimental results together

*Explore alternative physical pictures and mathematical descriptions without becoming wedded to any particular one

*Thermodynamic and other macroscopic arguments have precedence over microscopic calculations

*Focus on physical understanding, not mathematical elegance, and use the simplest possible mathematical description

*Keep up with new developments in theoretical techniques—for one of these may prove useful

*Decide on a model Hamiltonian or wave-function as the penultimate, not the first, step toward a solution

*Choose the right collaborators

*DON’T GIVE UP: Stay with the problem until it is solved

Question: Would BCS have ever come to pass if John had stayed at Bell Labs or gone anywhere other than UIUC?