BCS in Russia: the end of 50's –early 60's

(Developing Quantum Field theory approach to superconductivity)

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I. Science in Russia and the West

BY 1957: PRACTICALLY NO OUTSIDE CONTACTS YET

Russian tradition of scientific schools; Landau school; Russian science developing independently; a self-sufficient world

II. Superfluid He^4: helium II

Below I apply to SC some ideas from the physics of liquid helium. Landau developed the theory of HeII; Bogolyubov solved the model of weekly interacting Bose gas.

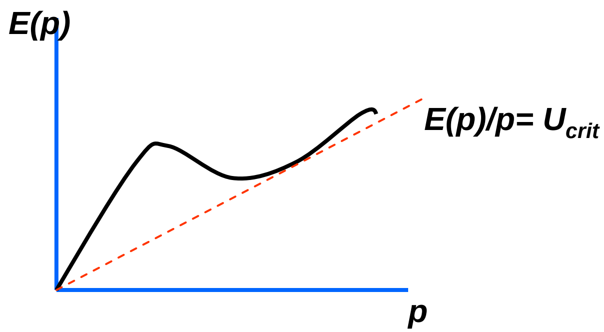
Lev Davidovich Landau

L. D. Landau, J. Phys. U.S.S.R.,**5**, 71(1940)

Nikolai Nikolaevich Bogolyubov

N. N. Bogolyubov, J. Phys. U.S.S.R.,**11**, 23 (1947)

Landau` views→ Let ***E(p)*** be the spectrum of *qp* excitations of He**II** : at T=0 there are no excitations as far as the liquid is at rest.

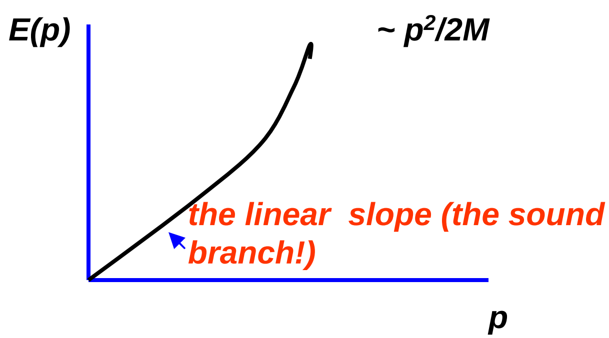


Assume that He**II** flows along a capillary (*U* is the velocity); viscosity as heating; energy cost of an excitation in the moving frame:

$$E_u(p)= E(p) + p.U$$ ***(The Galileo`s relativity principle)***

may become negative at large enough U. *The critical velocity, U_{crit}* ,corresponds to the moment when the line *p.U* first touches the *E(p)*-line: *U=E(p)/p,*

Bogolyubov solved the model of the Bose-gas with a weak repulsive interaction and obtained for the spectrum the linear slope at small p:



!!! the number of particles, N_0, in the Bose-condensate is macroscopically large (a finite density, $n_0=N_0 / V$),
the matrix elements for the condensate particles ARE also macroscopically large:
$(N-1| a_0| N) \sim (N+1 |a_0^+ | N) \sim N_0^{1/2}$

Non-diagonal matrix elements as the order parameter for HeII below T_{cond}!

III. The BCS –ideas recognized in Russia

a) The gapped spectrum [Landau criterion!]; The retarded *e-ph* attraction [J. Bardeen and D. Pines, 1955]

b) Cooper instability as the qualitative idea phenomenon, capable to explain why SC is so widespread among metals, and why T_c is low [the Cooper logarithm/exponent for Tc!]

c) Strong anisotropy of the Fermi surfaces in metals→ no "great expectations" as to the experiment (actually, as we know, the agreement was remarkable good for the isotropic model! [the Hebel-Slichter peak, 1959])

d) The theory's consistency and beauty:
Bogolyubov: the canonical transformation, variation of the Free Energy expressed in new *qp* for clean SC
Gor'kov: formulation in terms of Quantum Fields Theory (diagrammatic approach), general

IV. Landau seminar

My interests: my thesis(1955) in Quantum Electrodynamics of scalar mesons; Hydrodynamics; Hell
Bogolyubov announces his SC theory and is invited to talk at the Landau Seminar (October 1957)
Landau's refuses to understand the *ad hoc* "principle of compensation of the most dangerous diagrams", insists on the physics behind it
Exhausted N. N. finally gives up and produces the Leon Cooper's paper
I realize that it is about a new bosonic degree of freedom that appears below T_c

V. About Quantum Field Theory Methods (QFT)

Extension of the Quantum Electrodynamics` methods for the theory of metals at T=0 looked rather straightforward →with the Fermi sea ground state taken as the "vacuum". In terms of Feynman diagrams in its most systematic form : [V.M. Galitski and A.B. Migdal, JETP**, 7,** 95 (1958)].
Much of activity already at the time!

→**Migdal`s theory of electron-phonon interactions [** A.B. Migdal,JETP,**7,** 996(1958)].
→**Landau microscopic derivation of the Fermi liquid theory [L.D. Landau, Zh.ETF(1958), in JETP, 8, 70 (1959)**

VI. Developing QFT approach for theory of SC

A) My first paper on SC

SOVIET PHYSICS JETP VOLUME 34(7), NUMBER 3 SEPTEMBER, 1958

ON THE ENERGY SPECTRUM OF SUPERCONDUCTORS

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A method is proposed, based on the mathematical apparatus of quantum field theory, for the calculation of the properties of a system of Fermi particles with attractive interaction.

The model (BCS) Hamiltonian:

$$\dot{H} = \int \left\{ -\left(\psi^{+} \frac{\Delta}{2m} \psi\right) + \frac{g}{2} \left(\psi^{+} \left(\psi^{+} \psi\right) \psi\right) \right\} d^3x$$

By making use of the commutation relations for the field operators:
[x=(t,$\mathbf{x}$)]

$$\{\psi_\alpha(\mathbf{x}),\ \psi_\beta^{+}(\mathbf{x}')\} = \delta_{\alpha\beta}\,\delta(\mathbf{x} - \mathbf{x}'),$$
$$\{\psi_\alpha(\mathbf{x}),\ \psi_\beta(\mathbf{x}')\} = \{\psi_\alpha^{+}(\mathbf{x}),\ \psi_\beta^{+}(\mathbf{x}')\} = 0.$$

one obtains equations of motion:

$$\{i\partial/\partial t + \Delta/2m\}\,\psi(x) - g\,(\psi^{+}(x)\,\psi(x))\,\psi(x) = 0,$$
$$\{i\partial/\partial t - \Delta/2m\}\,\psi^{+}(x) + g\psi^{+}(x)\,(\psi^{+}(x)\,\psi(x)) = 0.$$

apply them to the x=(t,r) argument of the Green function:

$$G_{\alpha\beta}(x - x') = -i\,\langle T\,(\psi_\alpha(x),\ \psi_\beta^{+}(x'))\rangle$$

→ **products of four field operators appear, needed to be decoupled**

→**for simplicity the interaction between particles was taken into account insofar as it enters into formation of the bound pairs!**

→→

A sort of "Bose condensation" of pairs takes place in the case in which the momentum of their motion as a whole is equal to zero, just as in a Bose gas such a condensation takes place by virtue of the statistics for the particles themselv[es]. This circumstance permits us to write down in a definite way the mean form $\langle T(\psi(x_1)\,\psi(x_2)\times\psi^+(x_3)\,\psi^+(x_4))\rangle$, which appears in the equations for $G(x-x')$ by virtue of (3).

For example, we have

$$\langle T(\psi_\alpha(x_1)\,\psi_\beta(x_2)\,\psi_\gamma^+(x_3)\,\psi_\delta^+(x_4))\rangle =$$

$$-\langle T(\psi_\alpha(x_1)\,\psi_\gamma^+(x_3))\rangle\langle T(\psi_\beta(x_2)\,\psi_\delta^+(x_4))\rangle$$

$$+\langle T(\psi_\alpha(x_1))\,\psi_\delta^+(x_4))\rangle\langle T(\psi_\beta(x_2)\,\psi_\gamma^+(x_3))\rangle \qquad ($$

$$+\langle N|T(\psi_\alpha(x_1)\,\psi_\beta(x_2))|N$$

$$+2\rangle\langle N+2|T(\psi_\gamma^+(x_3)\,\psi_\delta^+(x_4))|N\rangle,$$

where $|N\rangle$ and $|N+2\rangle$ are the ground states of the system with numbers of particles N and $N+$ 2. The quantity → is of the order of density of pairs

To account for the non-diagonal character, take the time derivative of an operator:

$$\frac{\partial}{\partial t}\langle N|\hat{A}(t)|N+2\rangle = i(E_N - E_{N+2})\langle N|\hat{A}(t)|N+2\rangle.$$

one arrives at

$$\langle N|T(\psi_\alpha(x)\psi_\beta(x'))|N+2\rangle = e^{-2i\mu t}F_{\alpha\beta}(x-x'),$$

$$\langle N+2|T(\psi_\alpha^+(x)\psi_\beta^+(x'))|N\rangle = e^{2i\mu t}F_{\alpha\beta}^+(x-x'). \qquad (6)$$

[→ Note the Josephson exponential factor!]

and to the equations for all Green functions:

$$\{i\partial/\partial t + \Delta/2m\}\hat{G}(x-x') - ig\hat{F}(0+)\hat{F}^+(x-x') = \delta(x-x'),$$

$$\{i\partial/\partial t - \Delta/2m - 2\mu\}\hat{F}^+(x-x') + ig\hat{F}^+(0+)\hat{G}(x-x') = 0.$$

Notations for the anomalous functions mean, for instance:

$$F_{\alpha\beta}(0+) = e^{2i\mu t}\langle\psi_\alpha(x)\psi_\beta(x)\rangle$$

→the exponent can be removed by changing variables from ***N*** to the chemical potential

$$\hat{F}^+(0+) = F^+\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = F^+\hat{I};\quad \hat{F}(0+) = \cdot F\hat{I},$$

$$\hat{I} \equiv -i\hat{\sigma}_y$$

self-consistency for Δ:

$$\Delta = gF(0+) = g\,(2\pi)^{-4}\int F^+(p\omega)\,d\omega\,d^3k \;\longrightarrow\; 1 = -\frac{g}{2(2\pi)^3}\int\frac{d^3k}{\sqrt{\xi_k^2+\Delta^2}}\,(|\xi|<\varkappa)$$

Summary of results:

$g\hat{F}(0+);\ g\hat{F}^{+}(0+)$ → the pair wave function **is** the order parameter

→the BCS gapped spectrum and the Free Energy reproduced in few lines

→self-consistency as the *Anzatz* for the gap; no need in any variation procedure

→invariance of the Hamiltonian with respect to the gauge transformations of the field operators

→at T=0: the straightforward generalization of the diagrammatic technique that now involves the anomalous Green functions

→an inconvenience: at finite *T* one needs to use the boundary condition for Green functions:

$$\mathrm{Re}\,G(\omega) = -\frac{1}{\pi}\int_{-\infty}^{+\infty}\coth\frac{x}{2T}\,\frac{\mathrm{Im}\,G(x)}{\omega - x}\,dx$$

[L.D. Landau, JETP, **7**, 182 (1958)]

B) Applying the theory to superconducting alloys; Search for QFT methods at finite *T* (1957-1959):

Electrodynamics and Thermodynamics of alloys: A.A. Abrikosov and L.P.Gor'kov, **(I)** [ZhETF(1958)] JETP,**8**, 1090 (1959); **(II)** *ibid.*,**9**, 220 (1959) (Abrikosov joined me after my paper).

In **(I)** the diagrammatic "cross-technique" for scattering of electrons on defects elaborated;
In **(II)** our newly created "Matsubara" technique was first applied to alloys at nonzero *T*

Among the first results for SC alloys:

All the Green functions, including the anomalous (*F, F⁺*), after averaging over impurities:

*F(t-t',**R**); F⁺(t-t',**R**)* → *F(t-t',**R**) exp(-**R**/l); F+(t-t',**R**) exp(-**R**/l)*

(*l*- the mean free path). The "gap" is proportional to *F,F+ at **R**=0* **!**

→ in the isotropic model ordinary defects **do not** affect *Tc* and thermodynamics
"the Anderson Theorem"- [also P.W. Anderson, J. Phys.Chem. Solids, **11**,26 (1959)]

C) QFT methods at non-zero T: the thermodynamic technique

T=0:

$$\exp[-i(\hat{H}-\mu\hat{N})t] = \exp[-i(\hat{H}_0-\mu\hat{N})t]\times\hat{S}(t)$$

$$\hat{S}(\infty) = \hat{T}_t \exp(-i\int_{-\infty}^{+\infty}\hat{H}_{int}(t)dt)$$

$$G(x,x') = \frac{-i<\hat{T}(\hat{\Psi}(x)\hat{\Psi}^+(x')\hat{S}(\infty)>}{<\hat{S}(\infty)>}$$

Diagrammatic technique: the Fourier integrals
$(t,\mathbf{r}) \to (\omega, p)$

Finite T: [T.Matsubara, Prog. Theor.Phys., **14**,351, (1955)] found the formal analogy

$$\exp[\frac{\mu\hat{N}-\hat{H}}{T}] = \exp[\frac{\mu\hat{N}-\hat{H}_0}{T}]\times\hat{S}(1/T)$$

$$\hat{S}(1/T) = \hat{T}_\tau \exp(-\int_0^{1/T}\hat{H}_{int}(\tau)d\tau)$$

$$\mathfrak{G}(1,2) = -\frac{<<\hat{T}(\hat{\Psi}(1)\hat{\Psi}^+)(2)\hat{S}>>}{<<\hat{S}>>}$$

A.A.Abrikosov, L.P. Gor`kov and I.E. Dzyaloshinskii, JETP,**9**, 636(1959)[1958]→ Imaginary frequency;Fourier series and the analytical continuation from the imaginary axis:

$$\mathfrak{G}(\tau) = T\sum_n e^{-i\omega_n\tau}\mathfrak{G}(\omega_n),\ \omega_n = \pi T n,$$

D) General Eqs in coordinate representation. GL equations:

[L.P. Gor'kov: **(I)** JETP,**9**,1364(1959); **(II)** *ibid.*,***10***,998(1960)]

(I) clean SC

$$\left\{i\omega_n + \frac{1}{2m}\left(\frac{\partial}{\partial \mathbf{r}} - ie\mathbf{A}(\mathbf{r})\right)^2 + \mu\right\}\mathfrak{G}_\omega(\mathbf{r},\mathbf{r}') + \Delta(\mathbf{r})\mathfrak{F}^+_\omega(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}')$$

$$\left\{-i\omega_n + \frac{1}{2m}\left(\frac{\partial}{\partial \mathbf{r}} + ie\mathbf{A}(\mathbf{r})\right)^2 + \mu\right\}\mathfrak{F}^+_\omega(\mathbf{r},\mathbf{r}') - \Delta^*(\mathbf{r})\mathfrak{G}_\omega(\mathbf{r},\mathbf{r}') = 0,$$

$$\Delta^*(\mathbf{r}) = gT\sum_n \mathfrak{F}^+_\omega(\mathbf{r},\mathbf{r}), \qquad \omega = \pi T(2n+1),$$

Applicable at all T<Tc

→Note the Gauge invariance: $A(r) \Rightarrow A(r) + grad\phi$; $\Delta(r) \Rightarrow \Delta(r)\exp(i2\phi(r))$

By expanding near $T_c >> |T_c - T|$ one obtains the microscopic GL equations:

$$\left\{\frac{1}{2m}\left(\frac{\partial}{\partial \mathbf{r}} - ie^*A(\mathbf{r})\right) + \frac{1}{\lambda}\left[\frac{T_c - T}{T_c} - \frac{2}{N}|\Psi(\mathbf{r})|^2\right]\right\}\Psi(\mathbf{r}) = 0,$$

$$\mathbf{j}(\mathbf{r}) = -\frac{ie^*}{2m}\left(\Psi^*\frac{\partial\Psi}{\partial\mathbf{r}} - \Psi\frac{\partial\Psi^*}{\partial\mathbf{r}}\right) - \frac{e^{*2}}{mc}\mathbf{A}|\Psi|^2,$$

where $\Psi(\mathbf{r}) = \Delta(\mathbf{r})\sqrt{7\zeta(3)N}/4\pi T_c$. and $\lambda = 7\zeta(3)\epsilon_F/12(\pi T_c)^2$

Note ***the double charge, e*=2e, of the Cooper pair !***

$$\varkappa = 3T_c(\pi/v)^{1/2}(c/ep_0)\sqrt{2/7\zeta(3)}.$$

From this microscopic form it follows that even pure metals may belong to the so-called second type (e.g.,Nb, V)

(II) SC alloys

[add V(r), an impurity potential, and “turn on” the “cross”-technique →

$$\left\{\frac{1}{2m}\left(\frac{\partial}{\partial \mathbf{r}} - 2ie\mathbf{A}(\mathbf{r})\right)^2 + \frac{1}{\lambda_\tau}\left[\frac{T_c - T}{T_c} - \frac{2}{N\chi(\rho)}|\phi(\mathbf{r})|^2\right]\right\}\phi(\mathbf{r}) = 0,$$

$$\mathbf{j}(\mathbf{r}) = \frac{ie}{m}\left(\phi\frac{\partial\phi^*}{\partial\mathbf{r}} - \phi^*\frac{\partial\phi}{\partial\mathbf{r}}\right) - \frac{4e^2}{mc}\mathbf{A}(\mathbf{r})|\phi|^2.$$

$$\phi(\mathbf{r}) = [\chi(\rho)\, 7\zeta(3)\, N/16\pi^2 T_c^2]^{1/2}\Delta(\mathbf{r}),$$

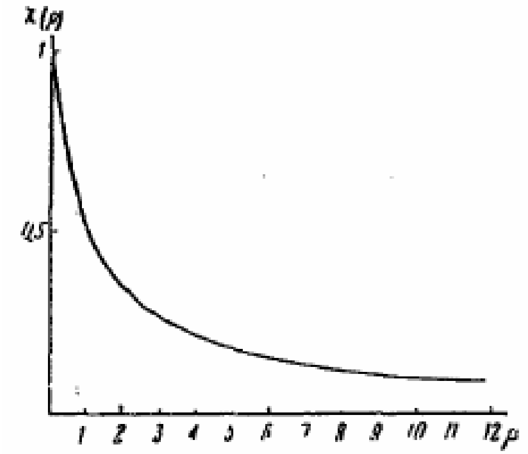
$$\lambda_\tau = \lambda_0\chi(\rho).$$

These are the GL eqs for alloys

$$\chi(\rho) = \frac{8}{7\zeta(3)}\sum_0^\infty \frac{1}{(2n+1)^2[2n+1+\rho]}$$

$$= \frac{8}{7\zeta(3)}\left(\frac{1}{\rho}\right)\left[\frac{\pi^2}{8} + \frac{1}{2\rho}\left\{\psi\left(\frac{1}{2}\right) - \psi\left(\frac{1}{2}+\rho\right)\right\}\right] \qquad (15)$$

(for $\rho \to 0$, $\chi(\rho) \to 1$, while for $\rho \to \infty$, $\chi(\rho) \approx \pi^2/7\zeta(3)\rho$). Here, $\rho = \frac{1}{2}\pi T_c \tau_{tr}$, $\psi(x)$ is the logarithmic derivative of the Γ function.



$$\delta = \delta_0/\sqrt{\chi(\rho)}.$$

$$\varkappa = \varkappa_0/\chi(\rho)$$

← The penetration depth and the GL “kappa”

In the “dirty” limit:

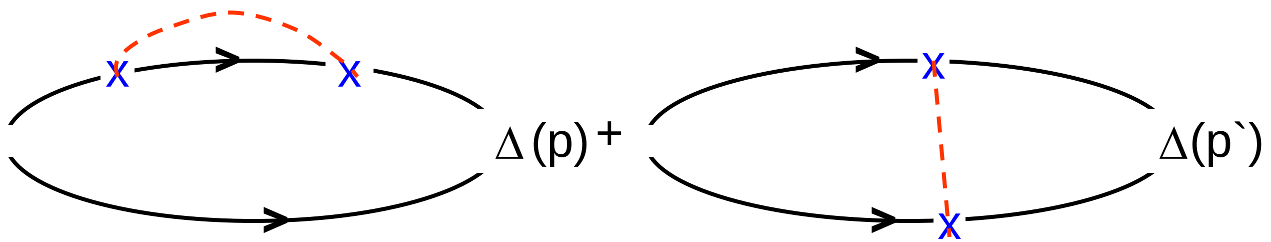
$$\varkappa = (ec\gamma^{1/2}/\sigma k\pi^3)\sqrt{21\zeta(3)/2\pi} = 0.065\, ec\gamma^{1/2}/\sigma k$$

Together with the GL and Abrikosov phenomenology these and others (films etc.) results constitute the GLAG- theory (Ginsburg-Landau-Abrikosov-Gor’kov)

(III) Paramagnetic Alloys and Gapless SC

[A.A. Abrikosov and L.P. Gor'kov, JETP,**12**, 1243(1960)]

I soon noticed that the Cooper instability could be affected by some impurities or an anisotropy



The two types of diagrams cancel each other for ordinary impurities in the isotropic case ["the Anderson theorem"!], but they do not if the two averages differ!

→ **the time-reversal symmetry is broken!**

$$V(p-p') \Rightarrow \tilde{V}(p-p') \times \hat{\sigma}.S$$

We obtained for *Tc*

$$\ln(T_{c0}/T_c) = \psi\left(\frac{1}{2} + \frac{h}{T_c\tau_s}\right) - \psi\left(\frac{1}{2}\right) \qquad (\ \psi(x) = \Gamma'(x)/\Gamma(x)\ )$$

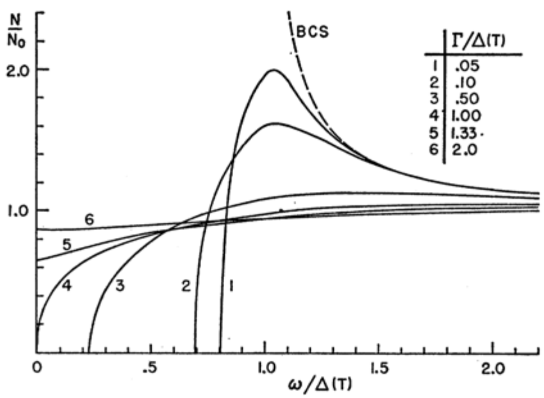
→T_c decreases to zero value at $\frac{1}{\tau_{scrit}} = \frac{\pi T_{co}}{2\gamma} \equiv \frac{\Delta_0}{2}$

However the gap in the energy spectrum closes first! $\omega_0 = [(e^{-\pi/4}\Delta_0)^{2/3} - (1/\tau_s)^{2/3}]^{3/2}$

While the *qp* momentum is not conserved in alloys, for ordinary defects the gap, i.e., the threshold for excitations, remains intact;
scattering on paramagnetic impurities results in a distribution over the pairs' binding energies.

$$N(E)/N_0(E_F) = \begin{cases} \Delta/(E^2-\Delta^2)^{1/2} \; ; & E>\Delta \\ 0; & \text{otherwise} \end{cases}$$

← the SC density of states for ordinary defects



← In paramagnetic alloys

[S. Skalski *et al.* Phys. Rev. 136 A,1500 (1964)]

(Note: the Landau criterion does not apply to alloys)

Collective effects (the non-zero order parameter) survive even when the *qp* excitation gap is zero

QFT methods have been applied to numerous problems

(instead of a summary)

→pairs` wave function as the order parameter
→simpler, automatically gauge- invariant; note that the *qp* have no fixed charge
→solves non-linear and inhomogeneous problems, [e.g.,H_{c2}***(0),*** L. Gor'kov (1960)], alloys
→easy generalization to anisotropic [V.Pokrovskii (1961)] and multi-band SC'tors
→no need in *qp`s* at calculations of thermodynamic and electrodynamic properties:

$$\Omega_s - \Omega_n = -\int d^3\mathbf{r} \int_0^g \frac{\delta g(\mathbf{r})}{g^2(\mathbf{r})} |\Delta(\mathbf{r})|^2.$$

(with $\Delta(\mathbf{r})$ obtained from equations for the Green functions [L. Gor'kov,1959])

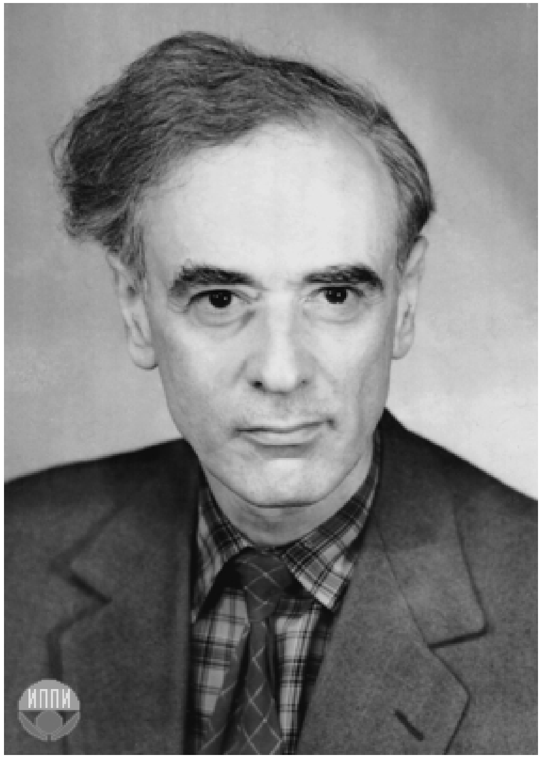
Applying the method further:

Eliashberg(1961, 1962) generalized the Migdal *e-ph* paper by applying the method of anomalous functions *F, F*$^+$ to the strong coupled SC'tors

For the BCS-like SC further simplifications turned out to be possible for the Gor'kov equations →non-linear equations for quasi-classical Green functions [G. Eilenberger, Z. Phys.**214**, 195 (1968)] → (sort of the master equations)

$$\begin{pmatrix} g(\omega) & f(\omega) \\ f^+(\omega) & \bar{g}(-\omega) \end{pmatrix} = \int \hat{G}(\omega, p) d\xi_p$$

The BCS model grew into the mighty theory of superconductivity to the considerable extent because of its formulation in terms of Green functions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