

**BCS everywhere else:
from Atoms and Nuclei to the Cosmos**

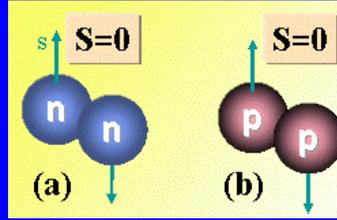
BCS@50
50th
ANNIVERSARY

**Gordon Baym
University of Illinois**

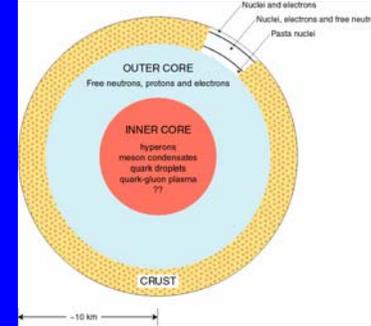
October 13, 2007

Wide applications of BCS beyond laboratory superconductors

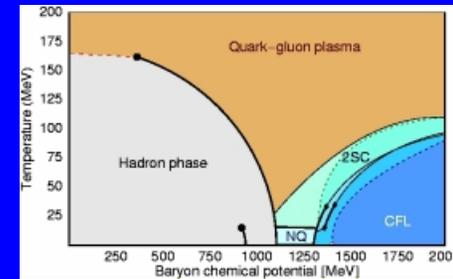
Pairing of nucleons in nuclei



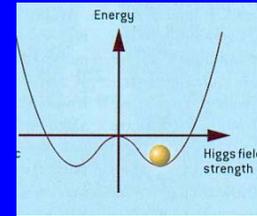
Neutron stars: pairing in neutron star matter



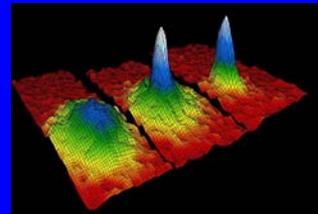
Pairing of quarks in degenerate quark-gluon plasmas



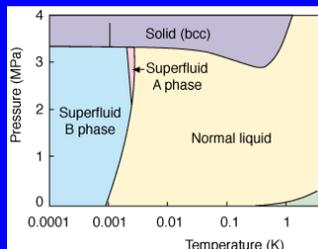
Elementary particle physics – broken symmetry



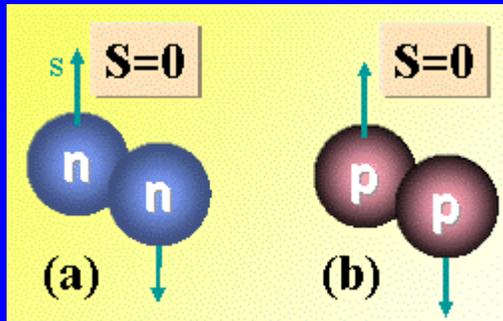
Cold fermionic atoms



Helium-3



BCS applied to nuclear systems - 1957



Pairing of even numbers of neutrons or protons outside closed shells

- *David Pines brings BCS to Niels Bohr's Institute in Copenhagen, Summer 1957, as BCS was being finished in Urbana.
- *Aage Bohr, Ben Mottelson and Pines (57) suggest BCS pairing in nuclei to explain energy gap in single particle spectrum – odd-even mass differences
- *Pairing gaps deduced from odd-even mass differences:
$$\Delta \simeq 12 A^{-1/2} \text{ MeV}$$
 for both protons and neutrons



B. Mottelson, M. Goeppert-Mayer, H. Jensen, Aa. Bohr
Conference on Nuclear Structure, Weizmann Institute,
Sept. 8-14, 1957

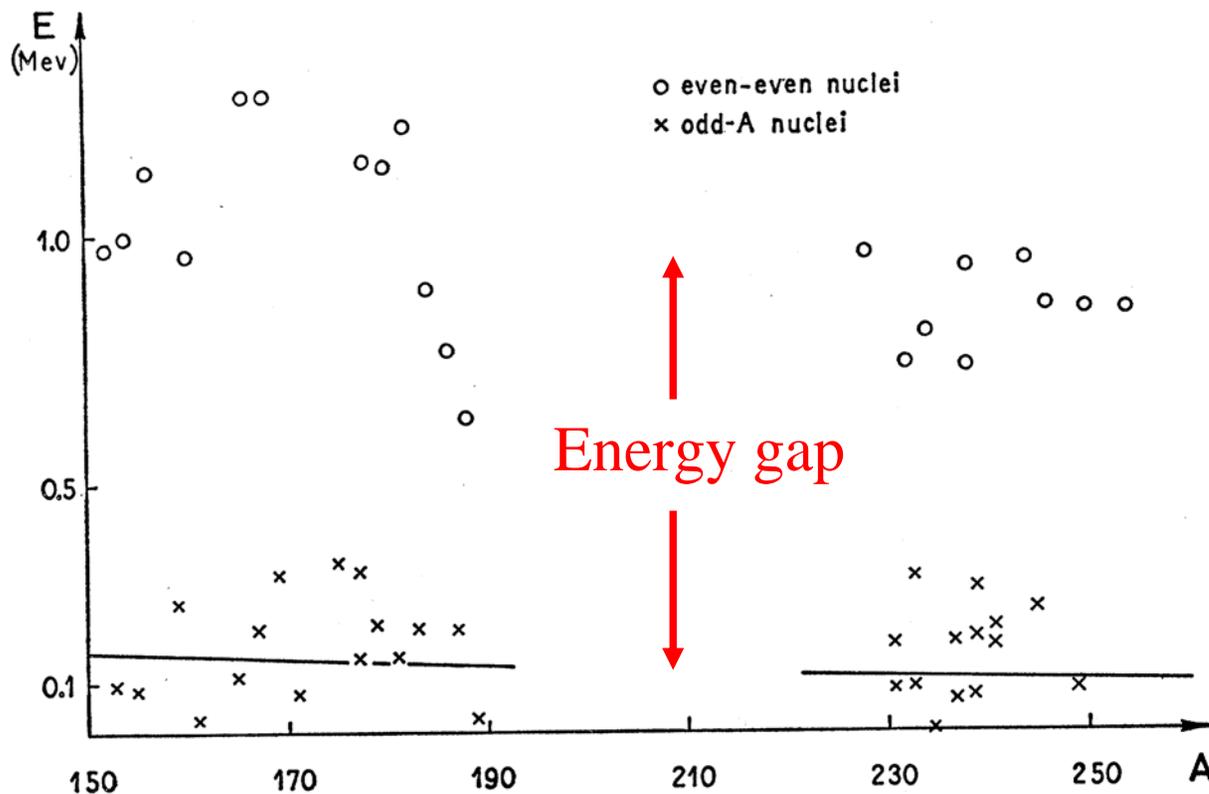
Energies of first excited states:

even-even (BCS paired) vs. odd A (unpaired) nuclei

FIG. 1. Energies of first excited intrinsic states in deformed nuclei, as a function of the mass number. The experimental data may be found in *Nuclear Data Cards* [National Research Council, Washington, D. C.] and detailed references will be contained in reference 1 above. The solid line gives the energy $\delta/2$ given by Eq. (1), and represents the average distance between intrinsic levels in the odd- A nuclei (see reference 1).

The figure contains all the available data for nuclei with $150 < A < 190$ and $228 < A$. In these regions the nuclei are known to possess nonspherical equilibrium shapes, as evidenced especially by the occurrence of rotational spectra (see, e.g., reference 2). One other such region has also been identified around $A=25$; in this latter region the available data on odd- A nuclei is still represented by Eq. (1), while the intrinsic excitations in the even-even nuclei in this region do not occur below 4 Mev.

We have not included in the figure the low lying $K=0$ states found in even-even nuclei around Ra and Th. These states appear to represent a collective odd-parity oscillation.



Possible Analogy between the Excitation Spectra of Nuclei and Those of the Superconducting Metallic State

A. BOHR, B. R. MOTTELSON, AND D. PINES*

Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark, and Nordisk Institut for Teoretisk Atomfysik, Copenhagen, Denmark

(Received January 7, 1958)

The evidence for an energy gap in the intrinsic excitation spectrum of nuclei is reviewed. A possible analogy between this effect and the energy gap observed in the electronic excitation of a superconducting metal is suggested.

THE nuclear structure exhibits many similarities with the electron structure of metals. In both cases, we are dealing with systems of fermions which may be characterized in first approximation in terms of independent particle motion. For instance, the statistical level density, at not too low excitation energies, is expected to resemble that of a Fermi gas. Still, in both systems, important correlations in the particle motion arise from the action of the forces between the particles and, in the metallic case, from the interaction with the lattice vibrations. These correlations decisively influence various specific properties of the system. We here wish to suggest a possible analogy between the correlation effects responsible for the energy gaps found in the excitation spectra of certain types of nuclei and those responsible for the observed energy gaps in superconducting metals.

proximately¹

$$\delta \approx 50A^{-1} \text{ Mev}, \quad (1)$$

where A is the number of particles in the nucleus.

If the intrinsic structure could be adequately described in terms of independent particle motion, we would expect, for even-even nuclei, the first intrinsic excitation to have on the average an energy $\frac{1}{2}\delta$, when we take into account the possibility of exciting neutrons as well as protons. Empirically, however, the first intrinsic excitation in heavy nuclei of the even-even type is usually observed at an energy of about 1 Mev (see Fig. 1). The only known examples of intrinsic excitations with appreciably smaller energy are the $K=0-$ bands which occur in special regions of nuclei, and which may possibly represent collective octupole vibrations.²

Rotational spectra of nuclei: $E = J^2 / 2I$, indicate moment of inertia, I , reduced from rigid body value, I_{cl} .

Reduction of moment of inertia due to BCS pairing = analog of Meissner effect. Detailed calculations by Migdal (1959).

Element	β [7]	x_p	x_n	$\left(\frac{J}{J_0}\right)_{\text{rect.}}$	$\left(\frac{J}{J_0}\right)_{\text{osc.}}$	$\left(\frac{J}{J_0}\right)_{\text{exper.}}$ [7]
Nd ¹⁵⁰	0.26	0.54	0.94	0.15	0.38	0.35
Sm ¹⁵²	0.24	0.65	1.02	0.17	0.43	0.38
Gd ¹⁵⁴	0.26	0.52	0.88	0.13	0.35	0.36
Gd ¹⁵⁶	0.33	0.87	1.37	0.22	0.57	0.48
Gd ¹⁵⁷	0.29	0.93	1.60	0.22	0.64	0.60
Dy ¹⁶²	0.30	0.84	1.43	0.23	0.57	0.50
Hf ¹⁷⁹	0.20	0.99	1.75	0.27	0.66	0.52
Os ¹⁸⁶	0.18	0.44	0.69	0.09	0.26	0.28
Th ²³⁰	0.22	0.63	0.95	0.15	0.40	0.43
Th ²³²	0.22	0.84	1.42	0.24	0.60	0.44
U ²³⁸	0.24	0.83	1.29	0.22	0.54	0.43

1.C: *Nuclear Physics* **13** (1959) 655—674; © North-Holland Publishing Co.
1.E.6 Not to be reproduced by photoprint or microfilm without written permission from the publisher.

SUPERFLUIDITY AND THE MOMENTS OF INERTIA OF NUCLEI

A. B. MIGDAL

Atomic Energy Institute of USSR, Academy of Sciences, Moscow

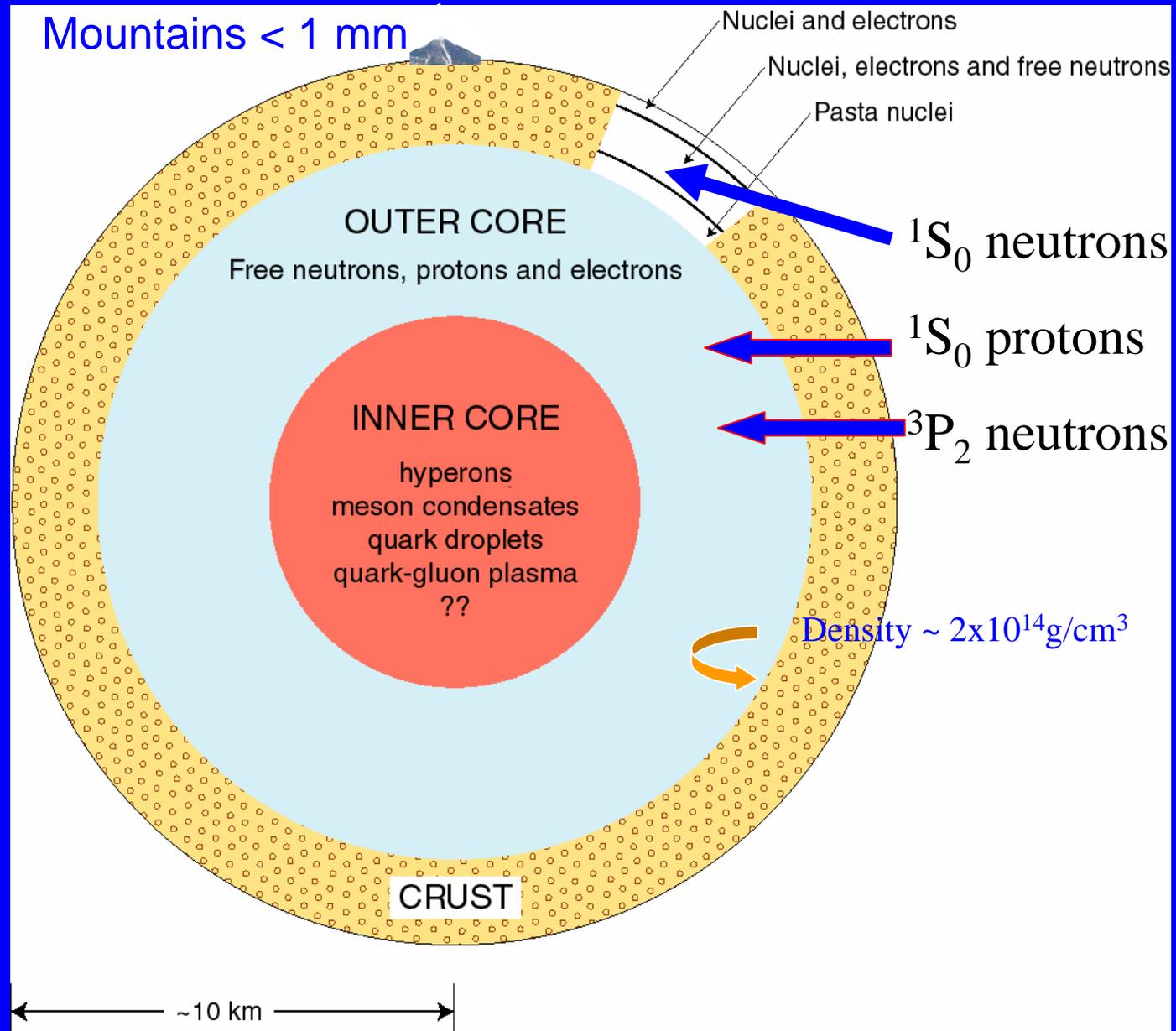
Received 11 April 1959

Abstract: A method is presented which permits one to study superfluidity in finite size systems. Moments of inertia are computed by this method in the quasi-classical approximation and satisfactory agreement with the observed values is obtained. The calculated increase of the moment of inertia upon transition from even to odd-mass nuclei and also the gyromagnetic ratio for rotating nuclei are in agreement with the experiments. These results thus confirm the assumption of superfluidity of nuclear matter.

BCS pairing of nucleons in neutron stars

Mass $\sim 1.4 M_{\text{sun}}$
Radius $\sim 10\text{-}12$ km
Temperature
 $\sim 10^6\text{-}10^9$ K

Surface gravity
 $\sim 10^{14}$ that of Earth
Surface binding
 $\sim 1/10 mc^2$



Neutron drip

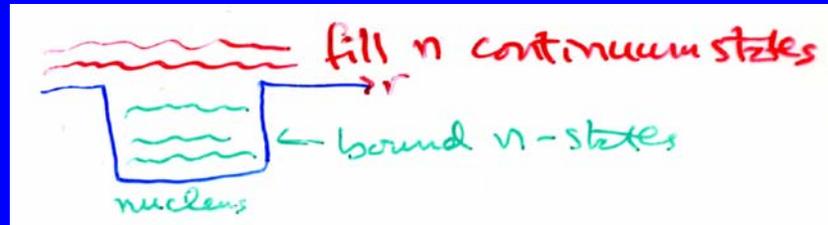


Beyond density $\rho_{\text{drip}} \sim 4.3 \times 10^{11} \text{ g/cm}^3$ neutron bound states in nuclei become filled through capture of high Fermi momentum electrons by protons:

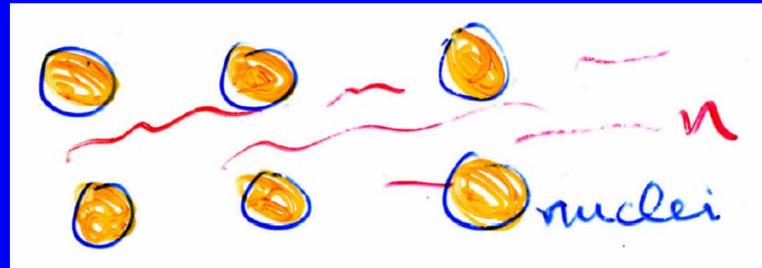


Further neutrons must go into continuum states.

Form degenerate neutron Fermi sea.



Neutrons in neutron sea are in equilibrium with those inside nucleus

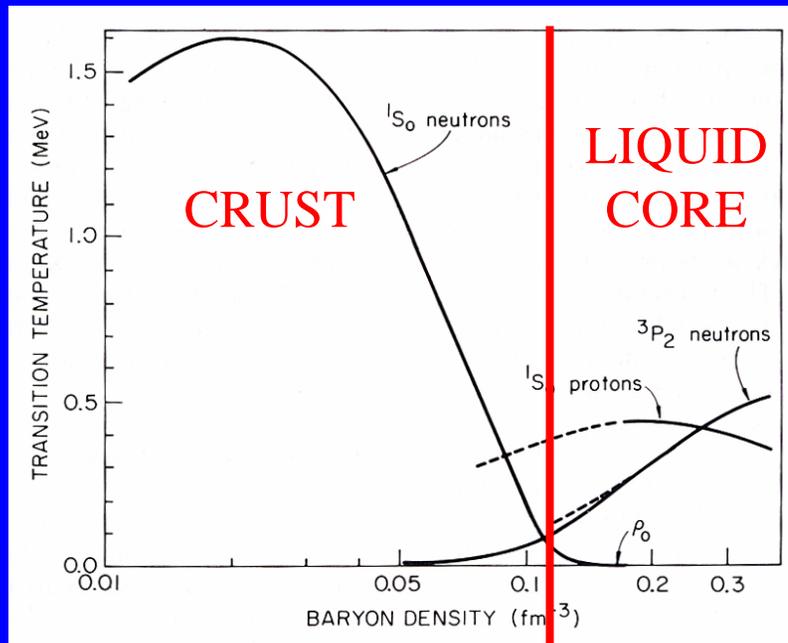


Protons never drip, but remain in bound states until nuclei merge into interior liquid.

Superfluidity of nuclear matter in neutrons stars

Migdal 1959, Ginzburg & Kirshnits 1964; Ruderman 1967; GB, Pines & Pethick, 1969

First estimates of pairing gaps based on scattering phase shifts



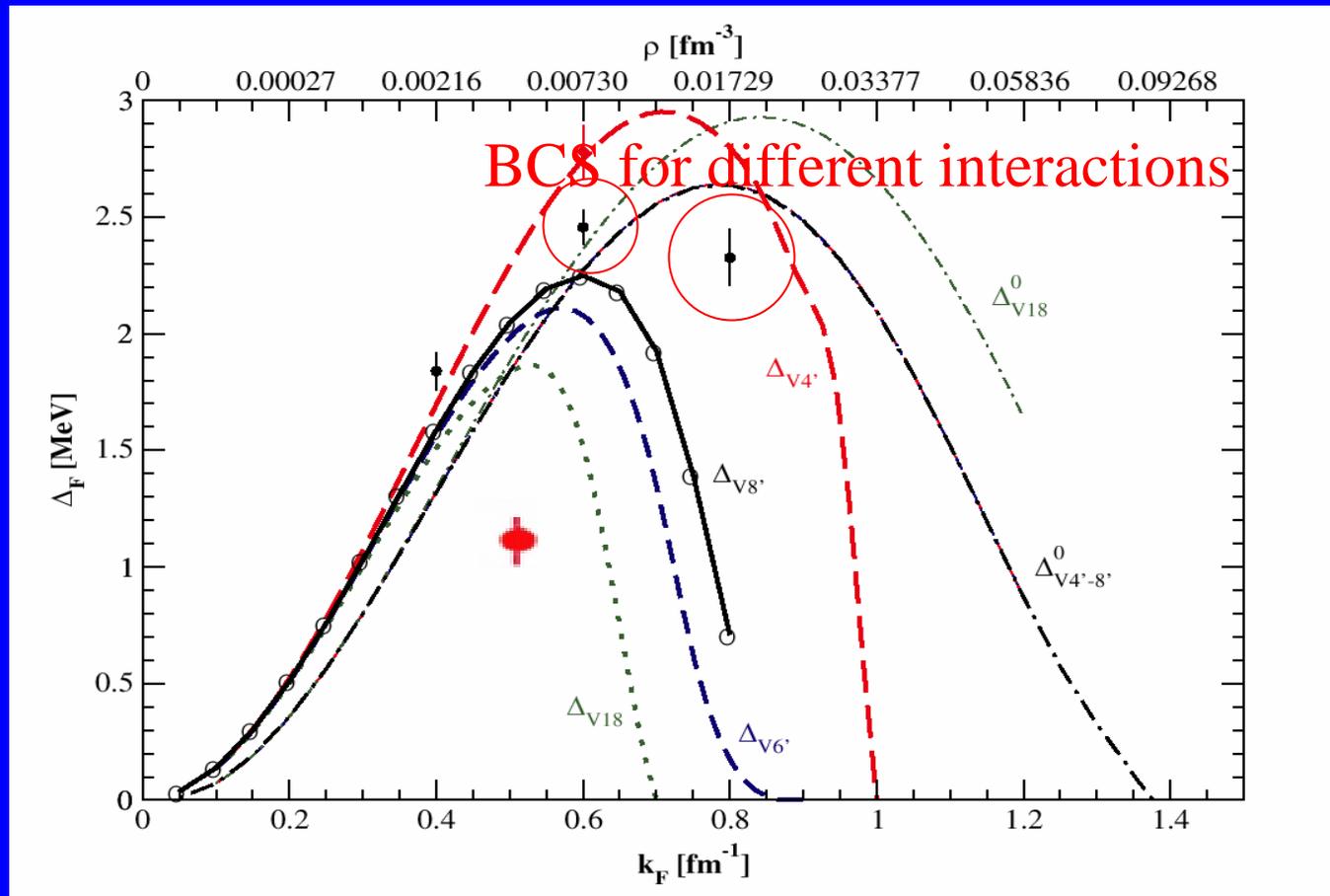
Neutron fluid in crust BCS-paired
in relative 1S_0 states

Neutron fluid in core 3P_2 paired
Proton fluid 1S_0 paired

n=Hoffberg et al. 1970, p=Chao et al. 1972

Quantum Monte Carlo (AFDMC) 1S_0 nn gap in crust:

Fabrocini et al, PRL 95, 192501 (2005)

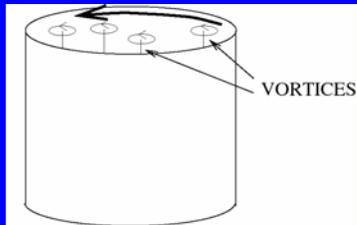


QMC (black points) close to standard BCS (upper curves)

Green's function Monte Carlo ● (Gezerlis 2007)

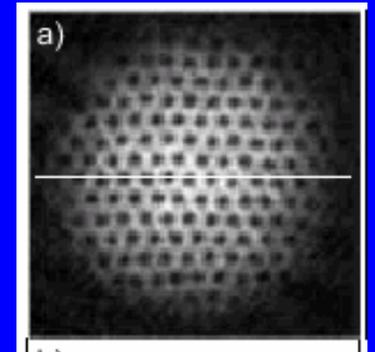
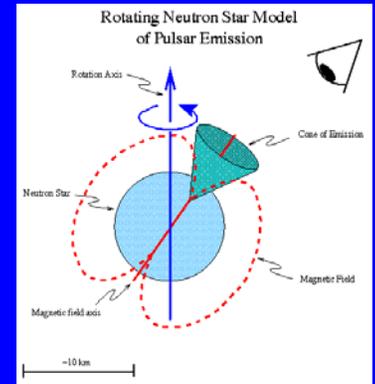
Rotating superfluid neutrons

Rotating superfluid threaded by triangular lattice of vortices parallel to stellar rotation axis



Bose-condensed ^{87}Rb atoms

Schweikhard et al., PRL92 040404 (2004)



Quantized circulation of superfluid velocity about vortex:

$$\oint_C \mathbf{v}_s \cdot d\mathbf{l} = \frac{2\pi\hbar}{2m_n}$$

Vortex core ~ 10 fm

Vortex separation $\sim 0.01P(\text{s})^{1/2}\text{cm}$; Vela contains $\sim 10^{17}$ vortices

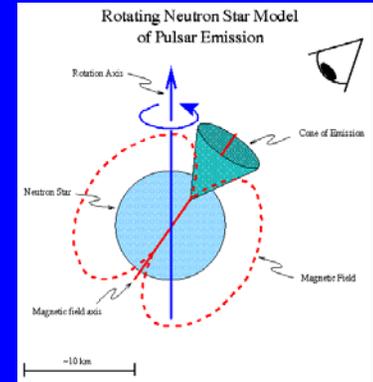
Angular momentum of vortex $=N\hbar(1-r^2/R^2)$ decreases as vortex moves outwards \Rightarrow **to spin down must move vortices outwards**

Superfluid spindown controlled by rate at which vortices can move against barriers, under dissipation

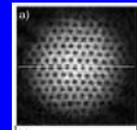
Superconducting protons in magnetic field

Even though superconductors expel magnetic flux, for magnetic field below critical value, flux diffusion times in neutron stars are \gg age of universe.

Proton superconductivity forms with field present.



Proton fluid threaded by triangular (Abrikosov) lattice of vortices parallel to magnetic field (for Type II superconductor)



Quantized magnetic flux per vortex:

$$\oint_C \mathbf{B} \cdot d\ell = \frac{2\pi\hbar c}{2e} = \phi_0 = 2 \times 10^{-7} \text{G}.$$

Vortex core ~ 10 fm,

$n_{\text{vort}} = B/\phi_0 \Rightarrow \text{spacing} \sim 5 \times 10^{-10} \text{ cm } (B / 10^{12} \text{G})^{-1/2}$

Pulsar glitches

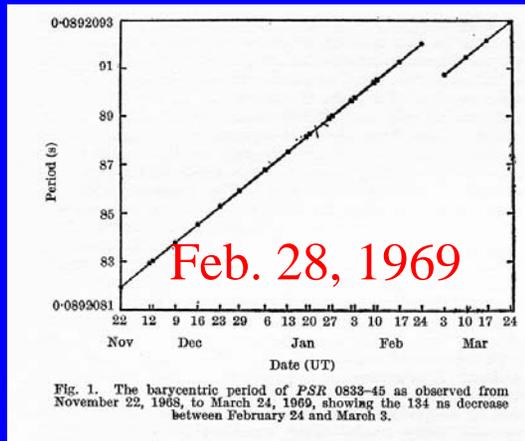
Sudden speedups in rotation period, relaxing back in days to years, with no significant change in pulsed electromagnetic emission
 ~ 90 glitches detected in ~ 30 pulsars

Vela (PSR0833-45) Period= $1/\Omega=0.089\text{sec}$

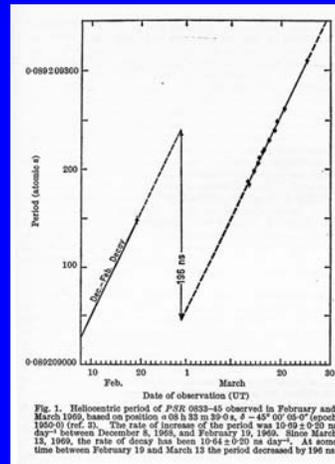
15 glitches since discovery in 1969

$\Delta\Omega/\Omega \sim 10^{-6}$ Largest = 3.14×10^{-6} on Jan. 16, 2000

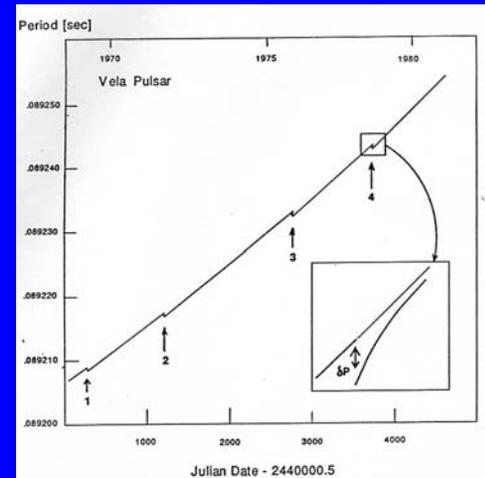
Moment of inertia $\sim 10^{45} \text{gcm}^2 \Rightarrow \Delta E_{\text{rot}} \sim 10^{43} \text{erg}$



Reichley and Downs, Nature 1969



Radhakrishnan and Manchester, Nature 1969



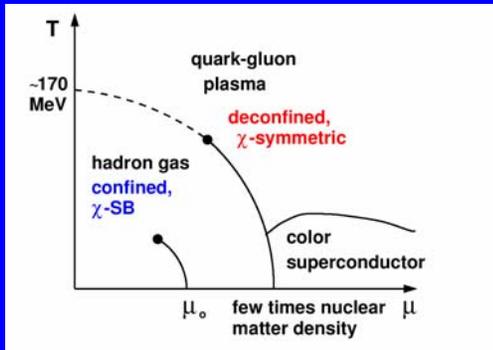
Crab (PSR0531+21) P = 0.033sec 14 glitches since 1969 $\Delta\Omega/\Omega \sim 10^{-9}$

Pairing in high energy nuclear/particle physics

Vacuum condensates: quark-antiquark pairing underlies

chiral $SU(3) \times SU(3)$ breaking of vacuum \Rightarrow

$$\langle \bar{q}q \rangle_{\text{vacuum}} \neq 0$$

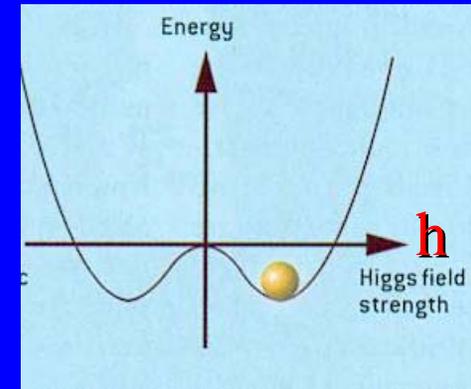


Experimental Bose-Einstein decondensation

Karsch & Laermann, hep-lat/0305025

Broken symmetry -- Particle masses via Higgs field

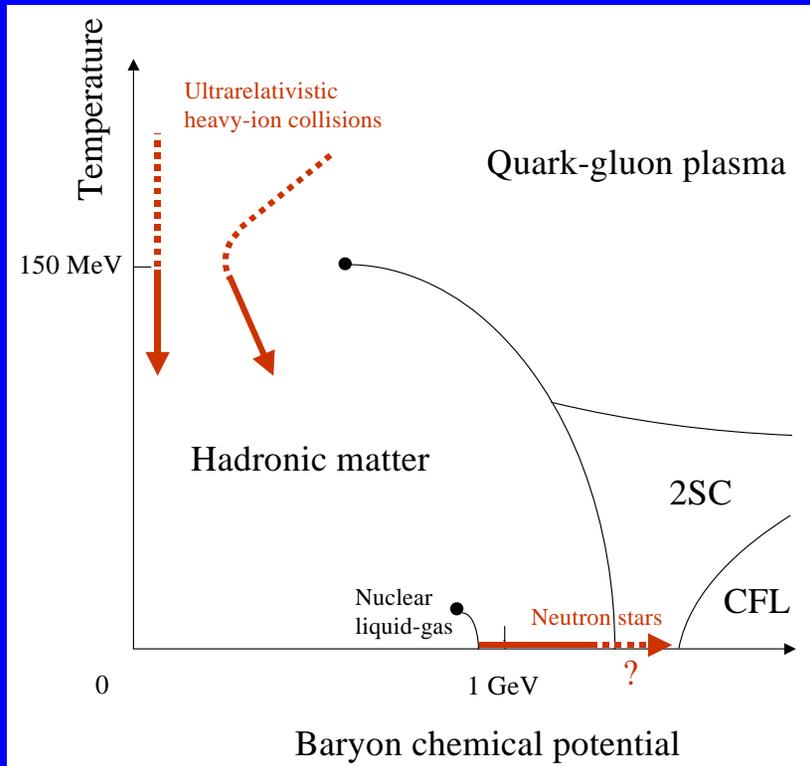
$$L_m = g h \psi^\dagger \gamma^0 \psi \Rightarrow g \langle h \rangle \psi^\dagger \gamma^0 \psi \Rightarrow m = g \langle h \rangle$$



BCS pairing of degenerate quark matter – color superconductivity

Color pairing in quark matter

Review: Alford, Rajagopal, Schaefer & Schmitt, arXiv:0709.4635

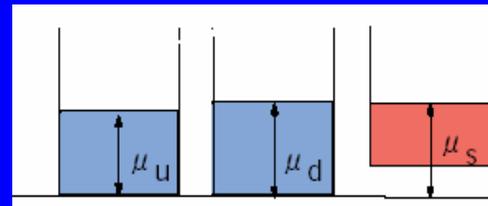


Superfluidity

condensate of paired quarks => superfluid baryon density (n_s)

Color Meissner effects

transverse color fields screened on spatial scale \sim London penetration depth $\sim (\mu/g^2 n_s)^{1/2}$



Two interesting phases:

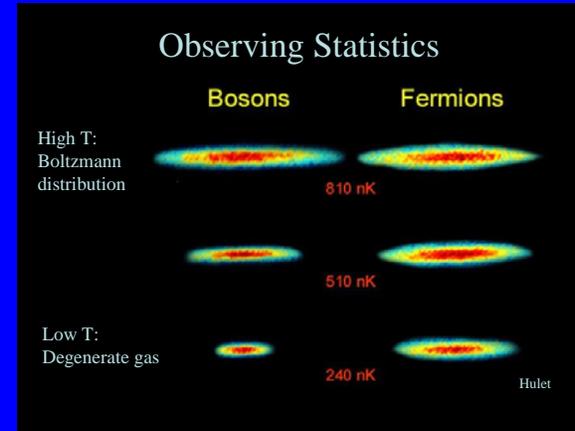
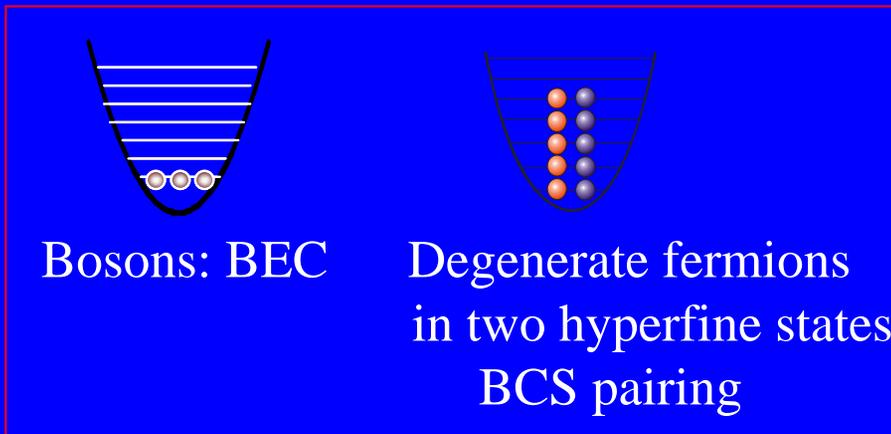
2SC (u,d)



Color-flavor locked (CFL) ($m_u = m_d = m_s$)



BCS paired fermions: a new superfluid



${}^7\text{Li}$ vs. ${}^6\text{Li}$

Produce trapped degenerate Fermi gases: ${}^6\text{Li}$, ${}^{40}\text{K}$
Increase attractive interaction with Feshbach resonance

At resonance have “unitary regime”: no length scale –
“resonance superfluidity”

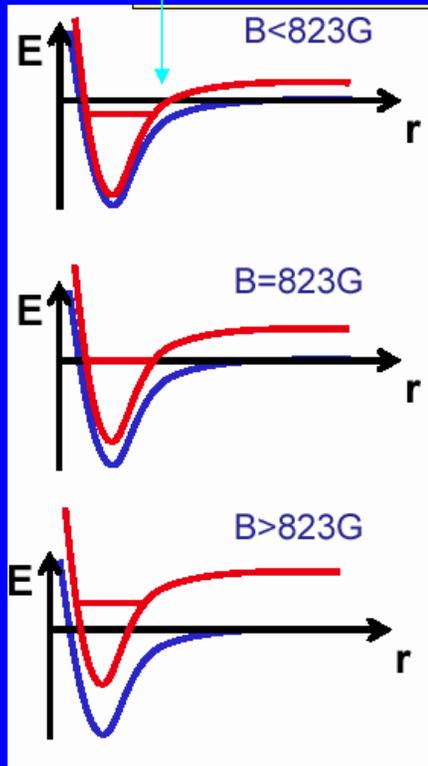
Experiments: JILA, MIT, Duke, Innsbruck, ...

Controlling the interparticle interaction

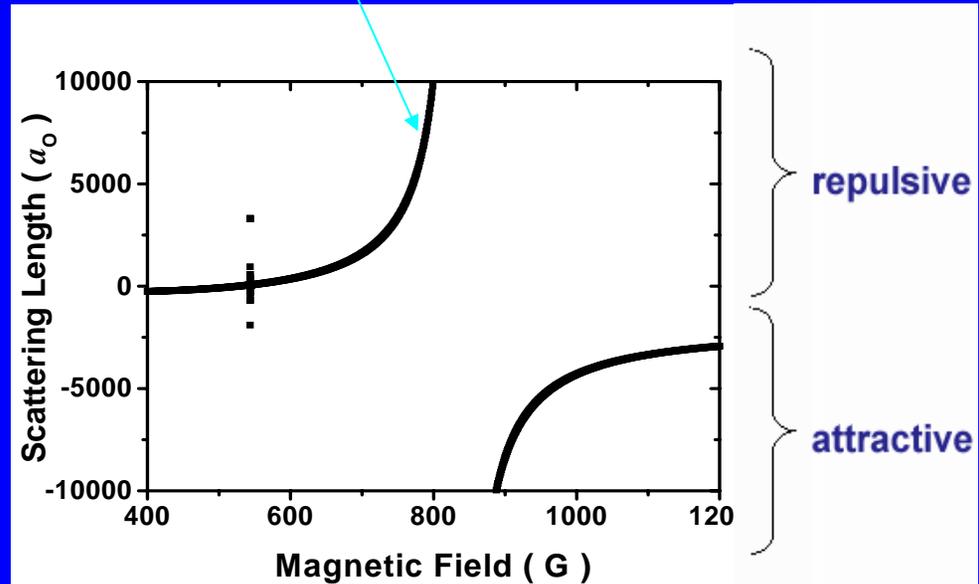
Effective interparticle interaction short range s-wave:

$$V(\mathbf{r}_1-\mathbf{r}_2) = (4\pi\hbar^2 a/m) \delta(\mathbf{r}_1-\mathbf{r}_2); \quad a = \text{s-wave atom-atom scattering length}$$

weakly bound molecule
in closed channel



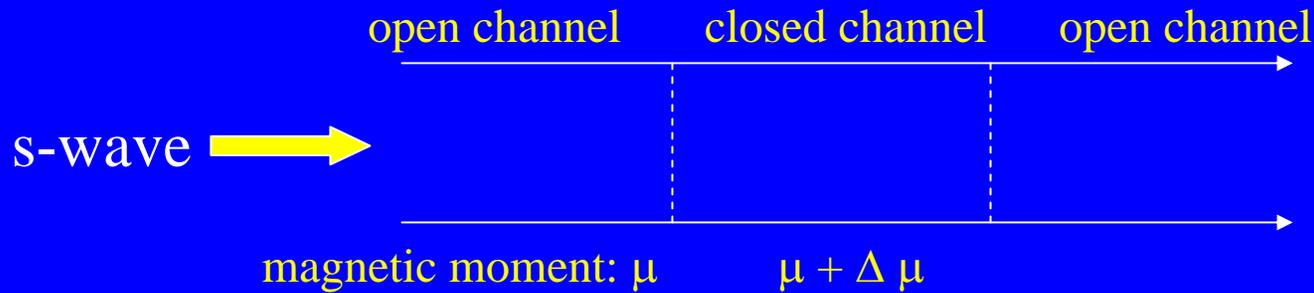
${}^6\text{Li}$



Broad resonance around 830 Gauss

Increasing magnetic field through resonance changes interactions from repulsive to attractive; very strong in neighborhood of resonance

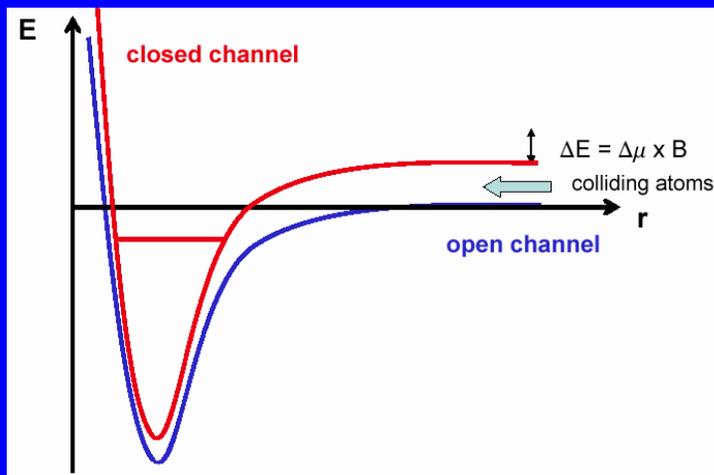
Feshbach resonance in atom-atom scattering



$$\text{Scattering amplitude} \propto \frac{|M|^2}{E_c - E_0}$$

$$E_c - E_0 \sim \Delta\mu B + \dots$$

Low energy scattering dominated by bound state closest to threshold



Adjusting magnetic field, B , causes level crossing and resonance, seen as divergence of s-wave scattering length, a :

$$a(B) = a_{bg} \left(1 - \frac{\Delta}{B - B_{Feshbach}} \right)$$

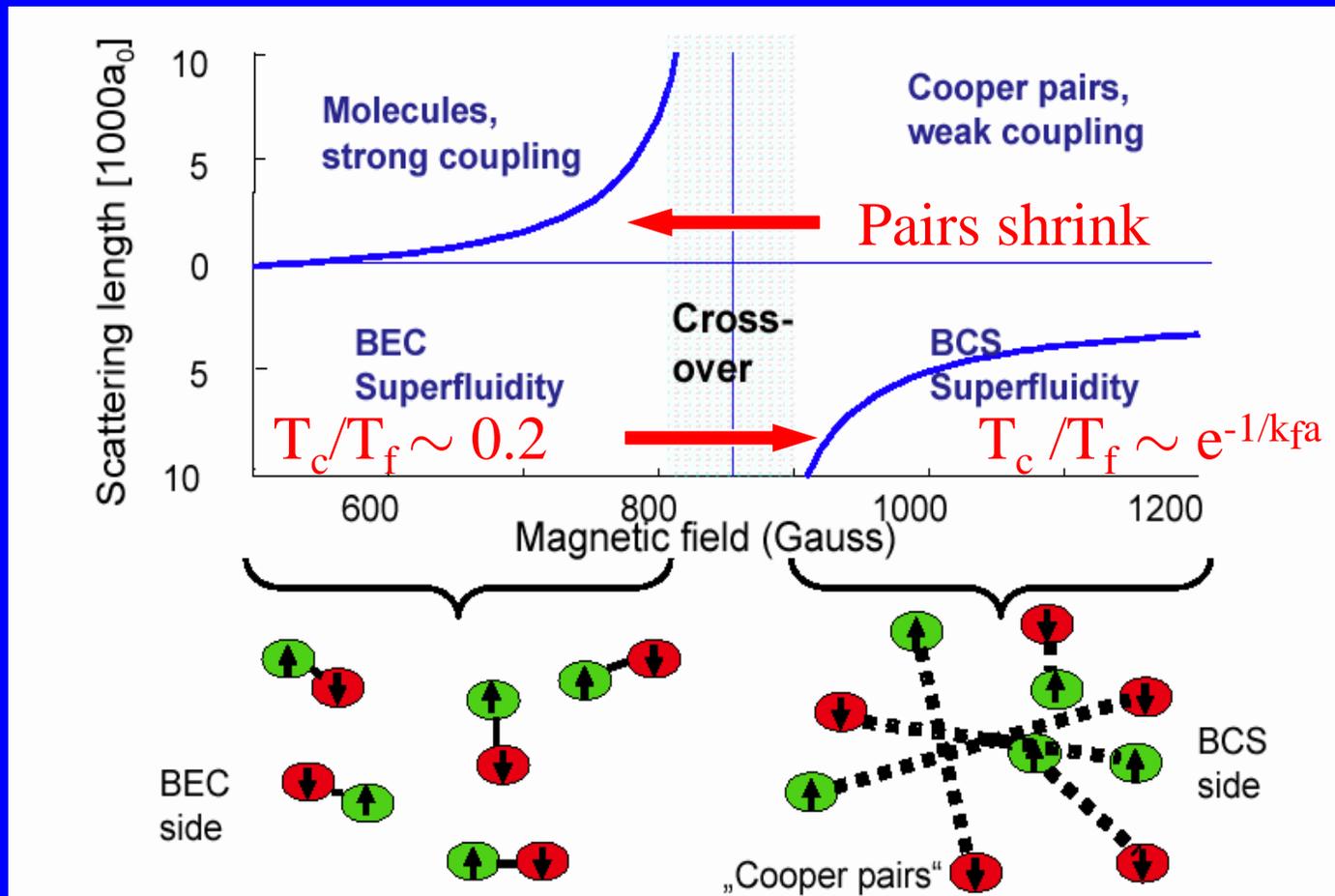
BEC-BCS crossover in Fermi systems

Continuously transform from molecules to Cooper pairs:

D.M. Eagles (1969)

A.J. Leggett, J. Phys. (Paris) C7, 19 (1980)

P. Nozières and S. Schmitt-Rink, J. Low Temp Phys. 59, 195 (1985)



⁶Li

Relation of Bose-Einstein condensation and BCS pairing?

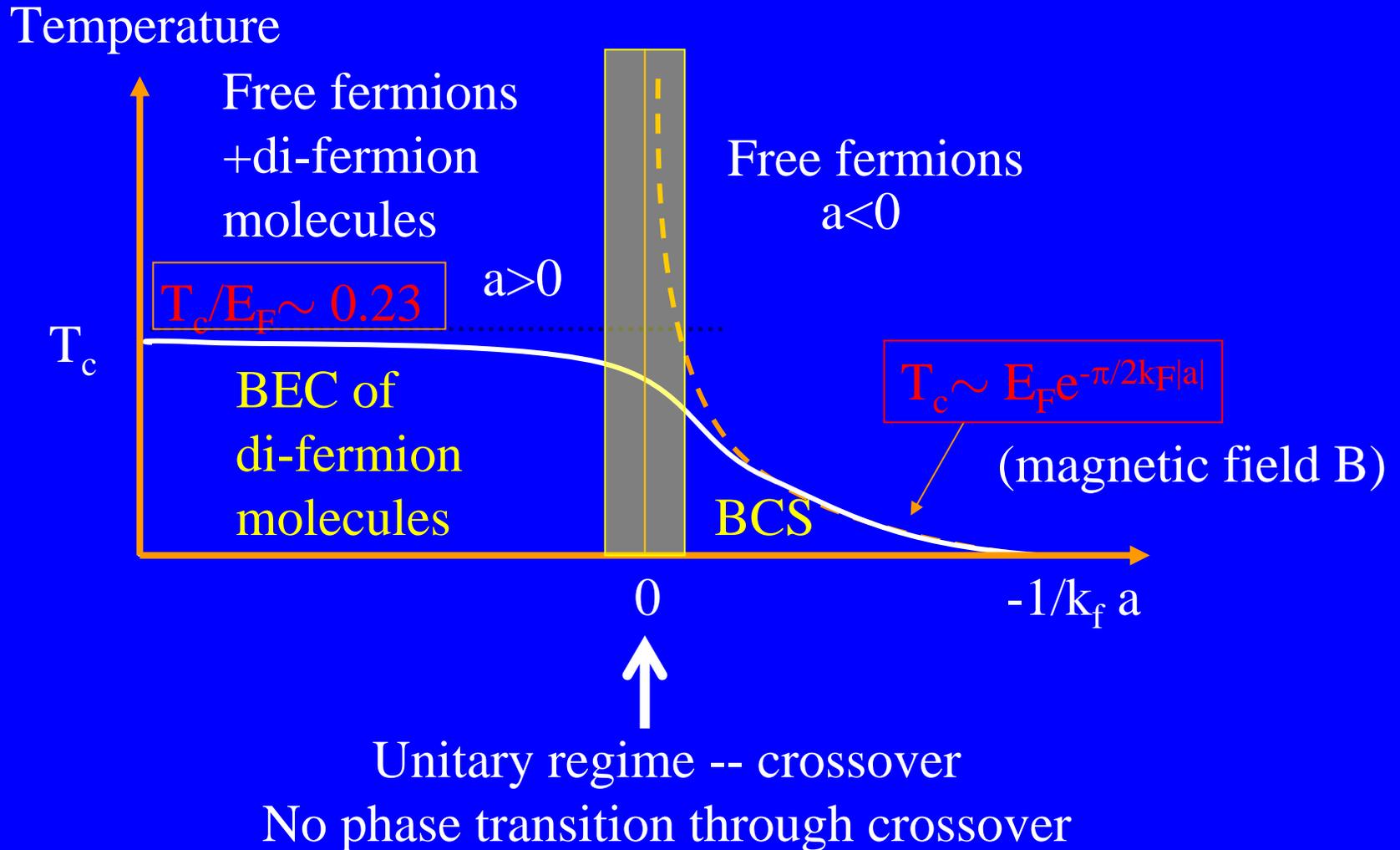
The two phenomena developed along quite different paths

“Our pairs are not localized ..., and our transition is not analogous to a Bose-Einstein condensation.”

BCS paper Oct. 1957

"We believe that there is no relation between actual superconductors and the superconducting properties of a perfect Bose-Einstein gas. The key point in our theory is that the virtual pairs all have the same net momentum. The reason is not Bose-Einstein statistics, but comes from the exclusion principle... ." *Bardeen to Dyson, 23 July 1957*

Phase diagram of cold fermions vs. interaction strength



Vortices in trapped Fermi gases: marker of superfluidity

*M.W. Zwierlein, J.R. Abo-Shaeer, A. Schirotzek, C.H. Schunck, and W. Ketterle,
Nature 435, 1047 (2005)*

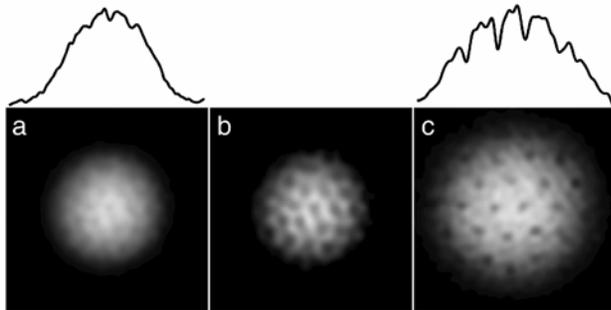


Fig. 1: Observation of a vortex lattice in a molecular condensate. (a) Fixed field. Stirring for 800 ms, followed by 400 ms of equilibration, and imaging after 12 ms time-of-flight all took place at 766 G. The vortex core depletion of the integrated density profile is barely 10%, as indicated by the 5- μm -wide cut on top. (b) Fourier-filter applied to (a) to accentuate the vortex contrast.

${}^6\text{Li}$

Resonance at $\sim 834\text{G}$

$B < 834\text{G} = \text{BEC}$

$B > 834\text{G} = \text{BCS}$

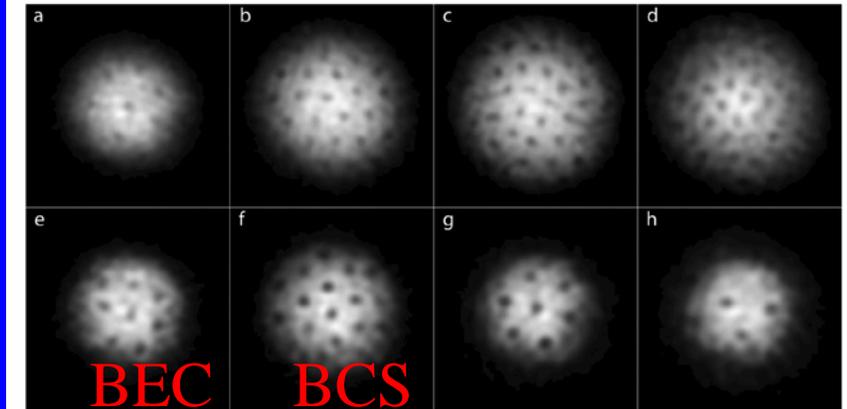
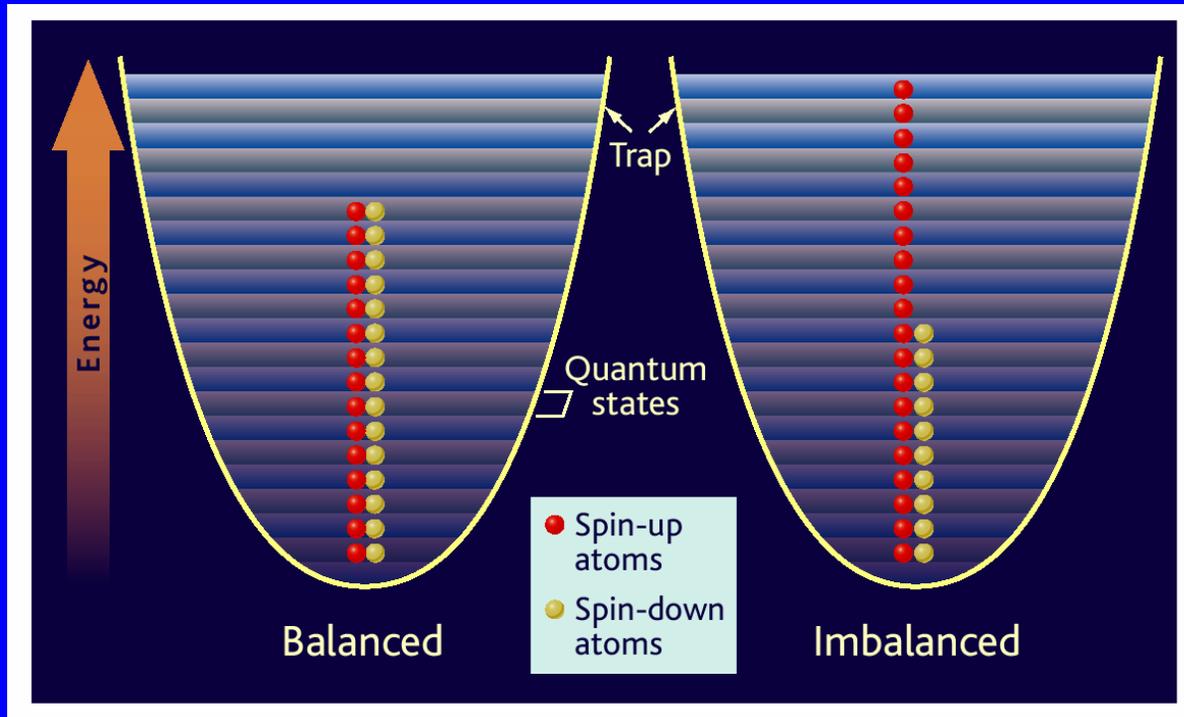


Fig. 2: Vortices in a strongly interacting gas of fermionic atoms on the BEC- and the BCS-side of the Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms (a) to 500 ms (b-h) followed by an equilibration time of 500 ms. After 2 ms of ballistic expansion, the magnetic field was ramped to 735 G for imaging (see text for details). The magnetic fields were (a) 740 G, (b) 766 G, (c) 792 G, (d) 812 G, (e) 833 G, (f) 843 G, (g) 853 G and (h) 863 G. The field of view of each image is $880\ \mu\text{m} \times 880\ \mu\text{m}$.

Superfluidity and pairing for unbalanced systems



Trapped atoms: change relative populations of two states by hand

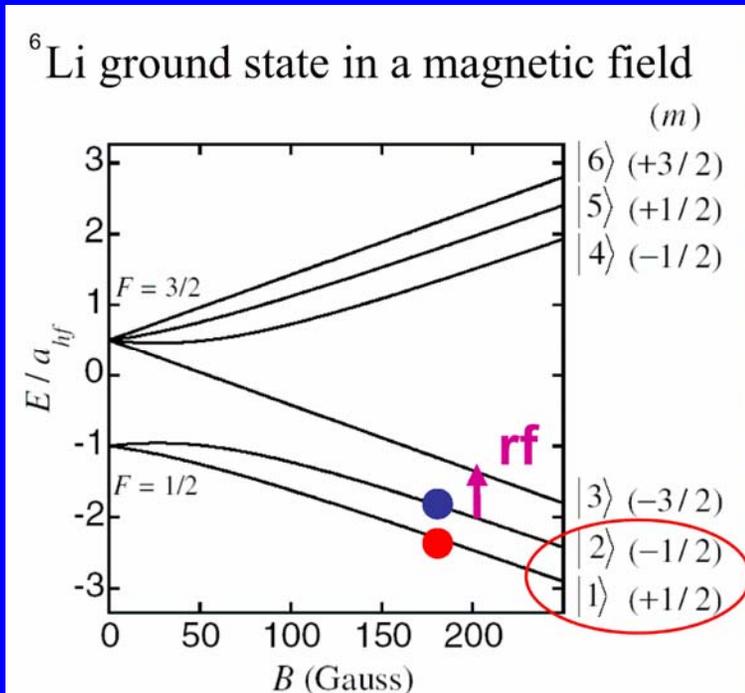
QGP: balance of strange (s) quarks to light (u,d) depends on

ratio of strange quark mass m_s to chemical potential μ (>0)

Experiments on ${}^6\text{Li}$ with imbalanced populations of two hyperfine states, $|1\rangle$ and $|2\rangle$

MIT: Zwierlein et al., *Science* 311, 492 (2006); *Nature* 442, 54 (2006);
Y. Shin et al., *arXiv: 0709.302*

Rice: G.B. Partridge, W. Li, R.I. Kamar, Y.A. Liao, and R.G. Hulet
Science 311, 503 (2006).



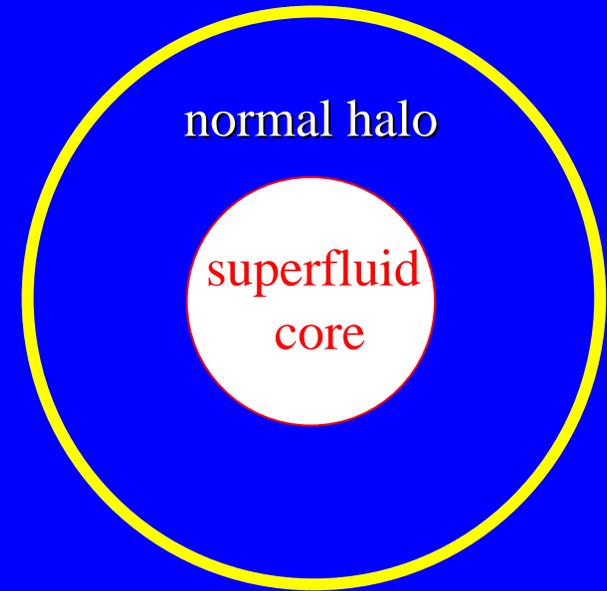
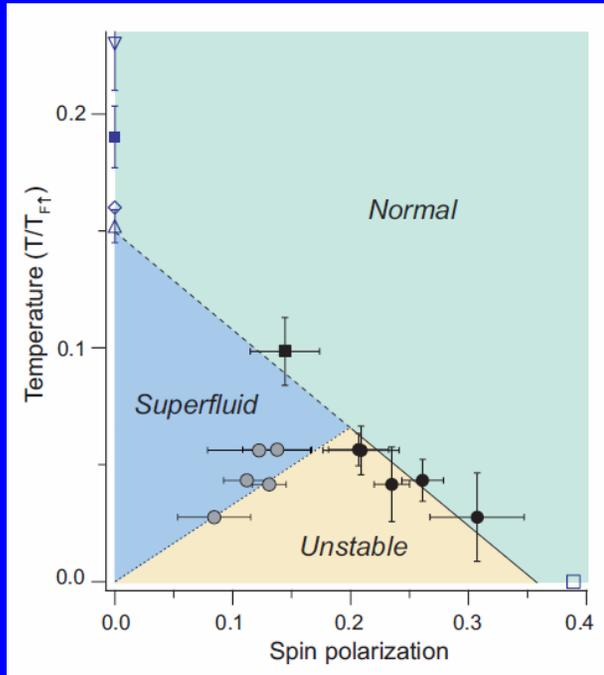
Fill trap with n_1 $|1\rangle$ atoms, and n_2 $|2\rangle$ atoms, with $n_1 > n_2$.

Study spatial distribution, and existence of superfluidity for varying $n_1:n_2$.

Phase diagram of trapped imbalanced Fermi gases

Y. Shin, C. H. Schunck, A. Schirotzek, & W. Ketterle, arXiv: 0709.3027

G.B. Partridge, W. Li, R.I. Kamar, Y.A. Liao, and R.G. Hulet, Science 311, 503 (2006).

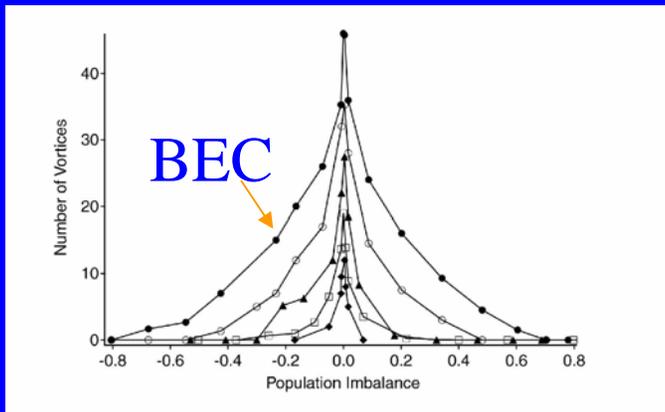
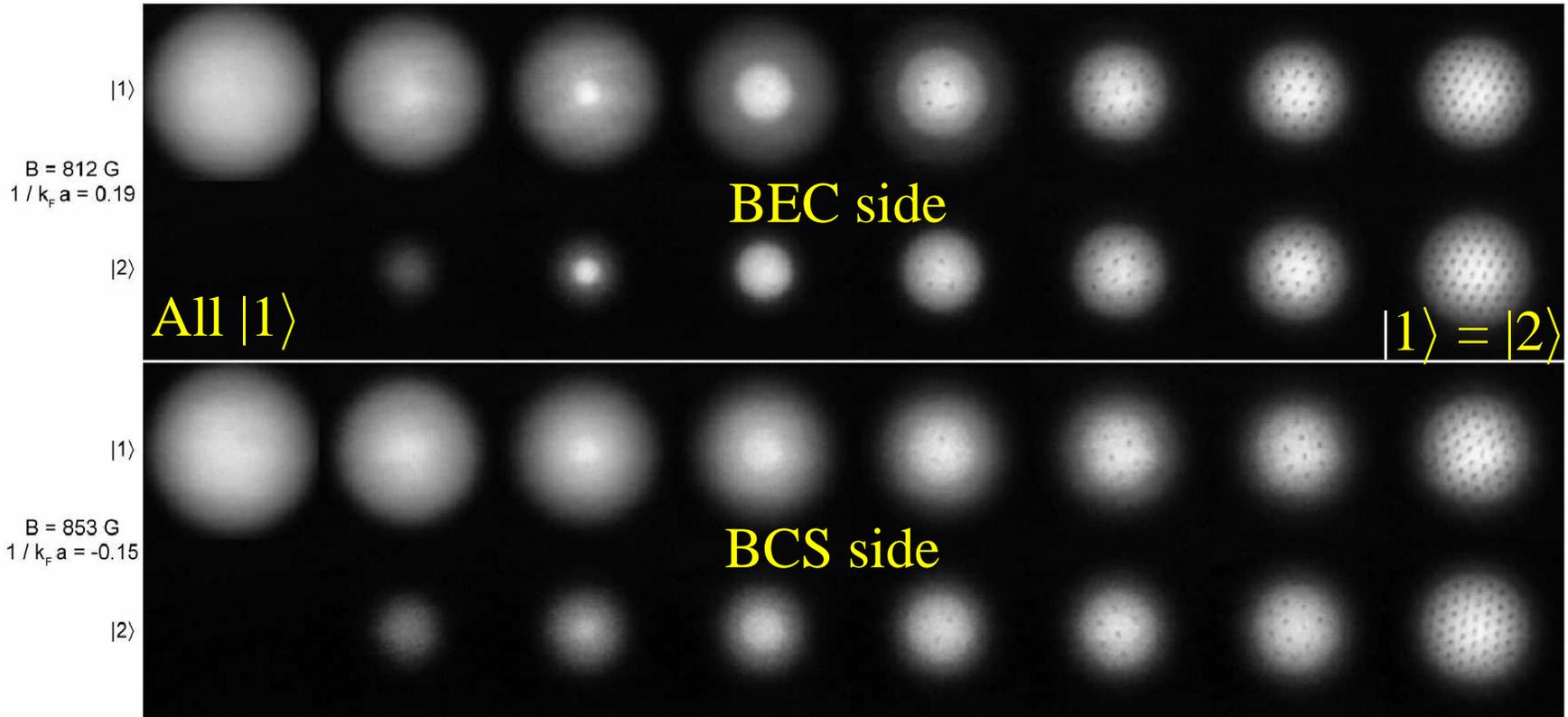


In trap geometry

Superfluid: second order transition to normal phase with increasing radius with gapless superfluid near boundary

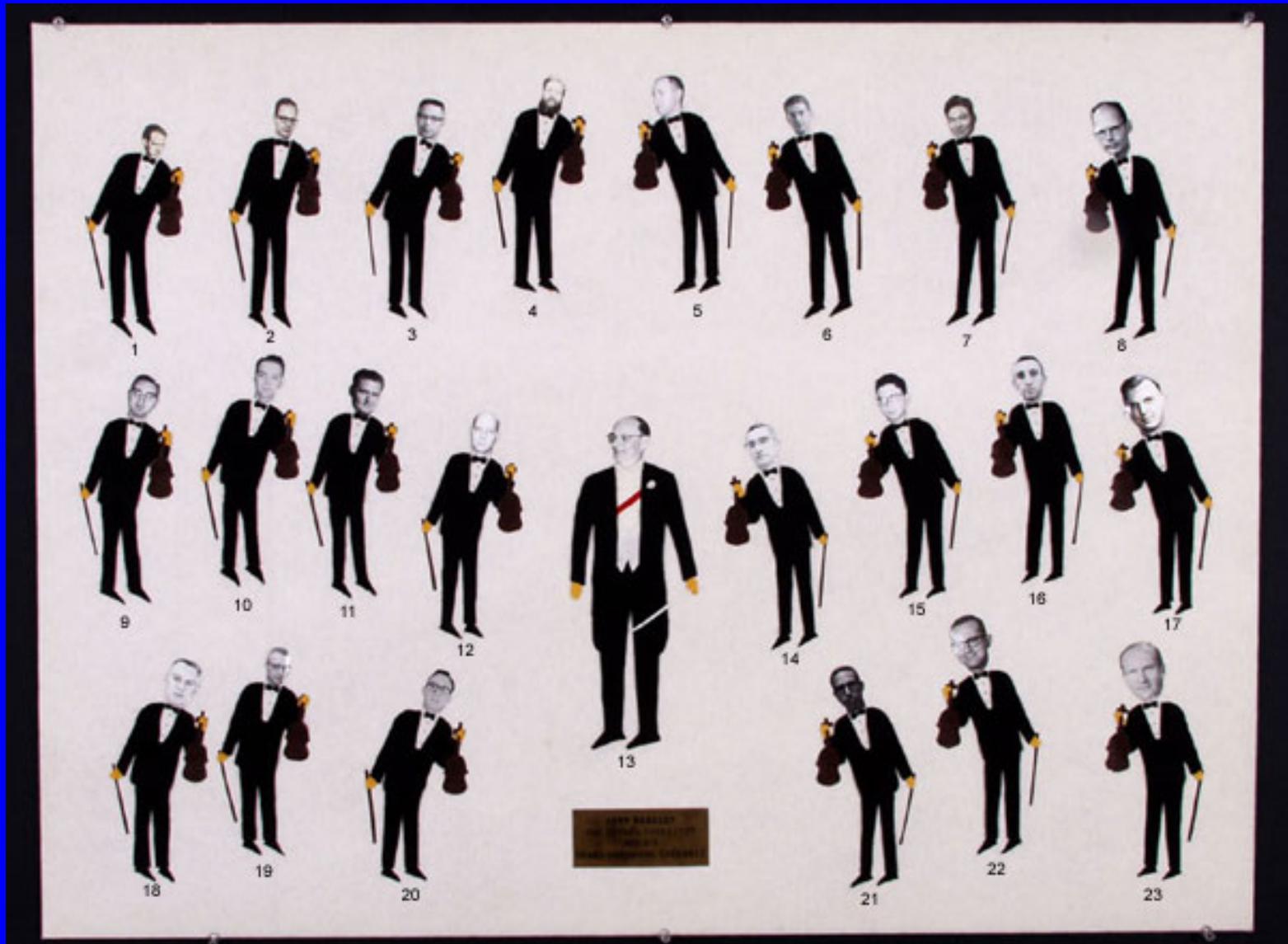
Unstable => phase separation: first order transition

Vortices in imbalanced paired fermions (MIT)



No. of vortices vs. population imbalance

John Bardeen – the Super Conductor



Bob & Anne Schrieffer

with his students, for his 60th birthday, 1968.