Fedora GNU/Linux; $\LaTeX$ 2ϵ; xfig

Color superconductivity in quark matter

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Outline

I Quarks at high density

Cooper pairing, color superconductivity

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III Compact stars

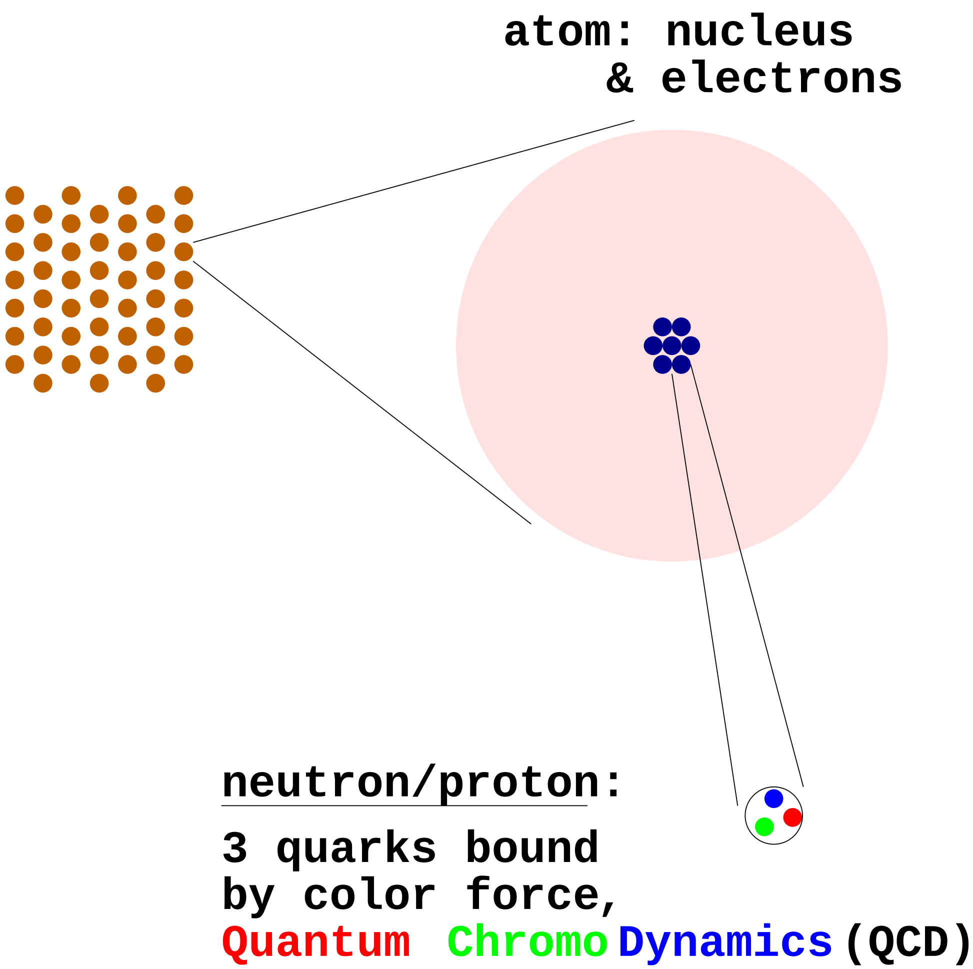
Transport properties and signatures

IV Looking to the future

M. Alford, K. Rajagopal, T. Schäfer, A. Schmitt, arXiv:0709.4635 (RMP review)

I. Quarks at high density

Quarks: Building blocks of matter



Quarks have color and flavor ("up" or "down")

proton: uud, uud, uud

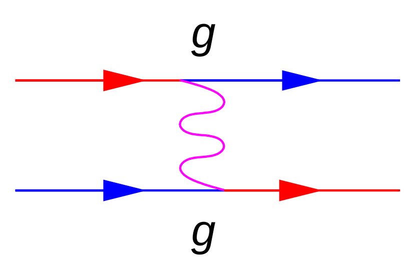
neutron: udd, udd, udd

Interactions between Quarks

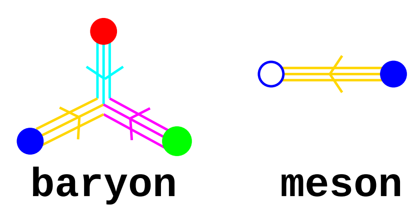
Dominant interaction between quarks is the strong interaction, described by $SU(3)$ "color" non-Abelian gauge theory (QCD).

Properties of QCD

- Short distances, $r \ll 1$ fm, asymptotically free: gauge coupling $g \ll 1$, single gluon exchange dominates, the theory is analytically tractable.



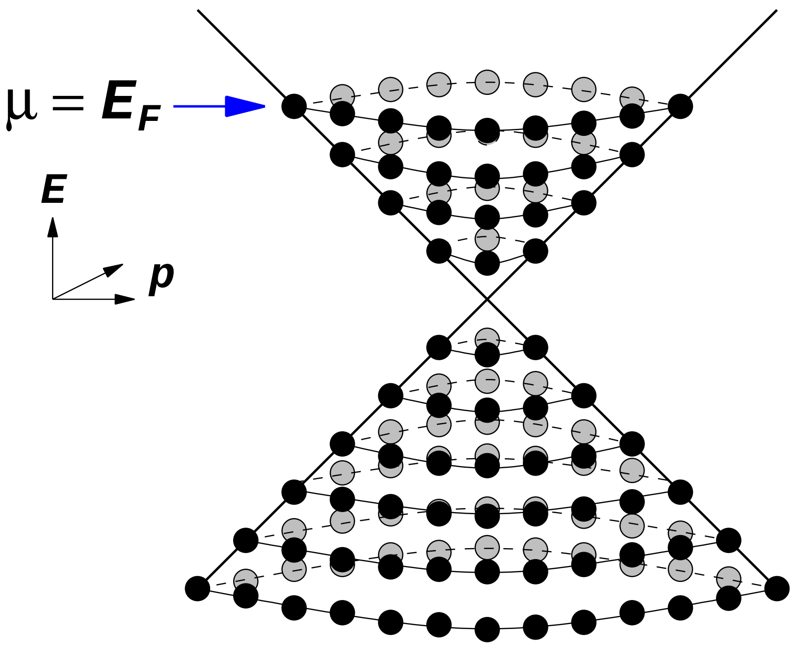
- Long distances $r > 1$ fm, QCD confines: color electric fields form flux tubes, only color-neutral states, baryons and mesons, exist.



Cooper pairing in quark matter: color superconductivity

At sufficiently high density and low temperature, there is a Fermi sea of almost free quarks.

Any attractive quark-quark interaction causes pairing instability of the Fermi surface: BCS mechanism of superconductivity.



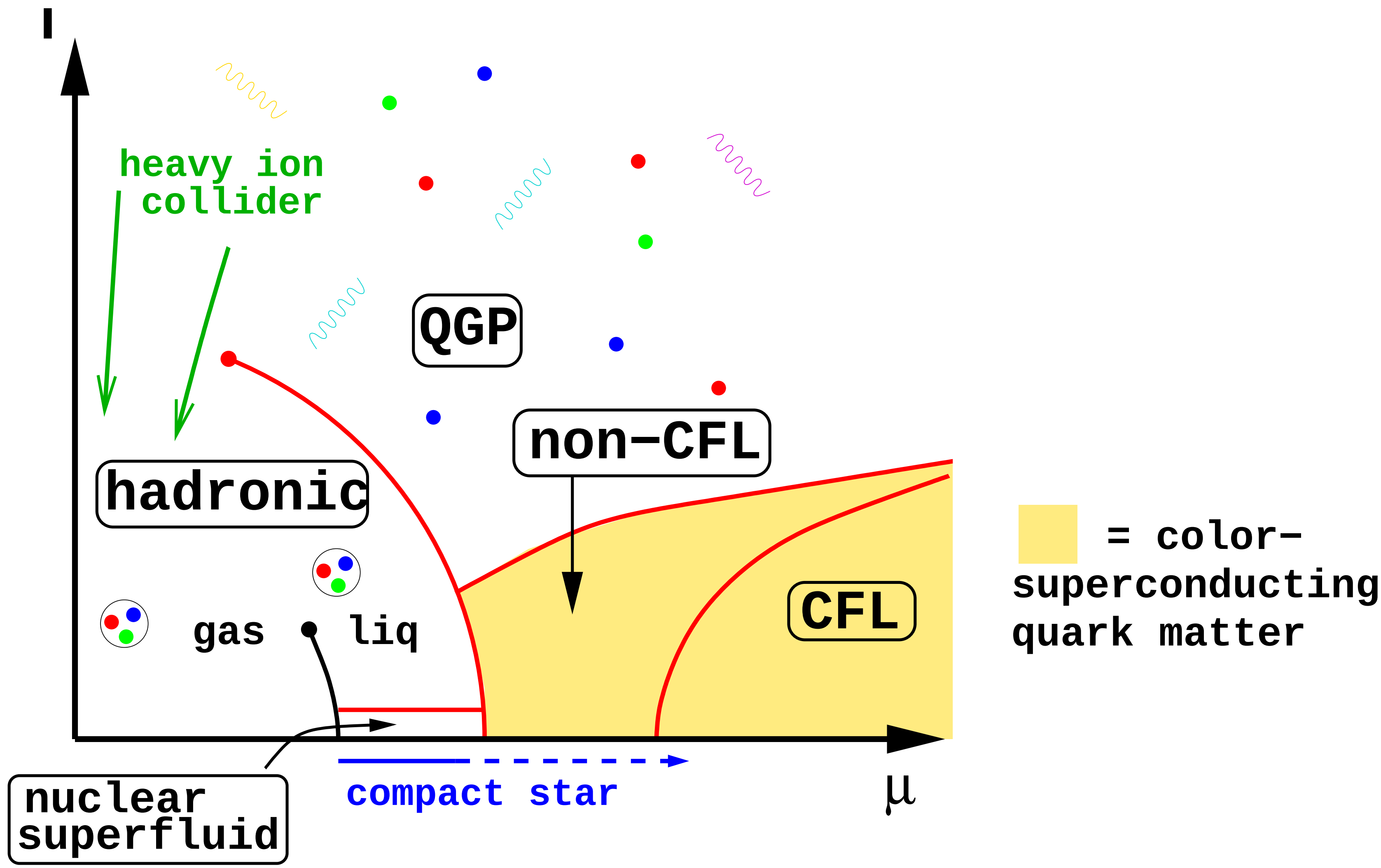
$$F = E - \mu N \qquad \frac{dF}{dN} = 0$$

QCD attractive quark-quark interaction in color-antisymmetric channel:

- single gluon exchange
- instanton interaction
- strong coupling: count flux tubes
- confinement is attraction

BCS in quark matter: Ivanenko and Kurdgelaidze, Lett. Nuovo Cim. IIS1 13 (1969).

Conjectured QCD phase diagram



heavy ion collisions: chiral critical point and first-order line

compact stars: color superconducting quark matter core

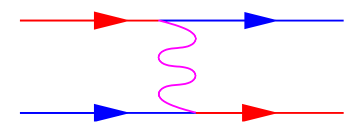
Handling QCD at high density

Lattice: "Sign problem"—negative probabilities

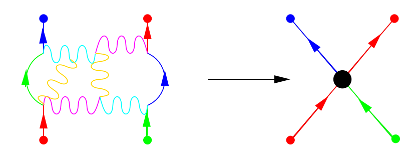
SUSY: Statistics crucial to quark Fermi surface

large N: Large corrections

pert: Applicable far beyond nuclear density.
Neglects confinement and instantons.



NJL: Model, applicable at low density.
Follows from instanton liquid model.



EFT: Effective field theory for lightest degrees of freedom.
"Parameterization of our ignorance": assume a phase, guess coefficients of interaction terms (or match to pert theory), obtain phenomenology.

II. Color superconducting phases

Quark Cooper pair: $\langle q^{\alpha}_{ia} q^{\beta}_{jb} \rangle$

color $\alpha, \beta = r, g, b$

flavor $i, j = u, d, s$

spin $a, b = \uparrow, \downarrow$

There is a 9×9 matrix of possible BCS pairing patterns!

The attractive channel is:

color antisymmetric

spin antisymmetric

$\Rightarrow$ flavor antisymmetric

So pairing between *different flavors* is favored.

Let's start with the most symmetric case, where all three flavors are massless.

3 massless flavors: Color-flavor locking (CFL)

Equal number of colors and flavors gives a special pairing pattern
(Alford, Rajagopal, Wilczek, hep-ph/9804403)

$$\langle q_i^\alpha q_j^\beta \rangle \sim \delta_i^\alpha \delta_j^\beta - \delta_j^\alpha \delta_i^\beta = \epsilon^{\alpha\beta n}\epsilon_{ijn}$$

color α, β
flavor i, j

This is invariant under equal and opposite rotations of color and (vector) flavor

$$SU(3)_{\text{color}} \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B \to \underbrace{SU(3)_{C+L+R}}_{\supset U(1)_{\tilde{Q}}} \times \mathbb{Z}_2$$

- Breaks chiral symmetry, but *not* by a $\langle \bar{q} q \rangle$ condensate.
- There need be no phase transition between the low and high density phases: ("quark-hadron continuity")
- Unbroken "rotated" electromagnetism, $\tilde{Q}$, photon-gluon mixture.

Color-flavor-locked ("CFL") quark pairing

$\tilde{Q}$	0	0	0	-1	$+1$	-1	$+1$	0	0
	u	d	s	d	u	s	u	s	d
u		Δ	Δ						
d	Δ		Δ						
s	Δ	Δ							
d					$-\Delta$				
u				$-\Delta$					
s							$-\Delta$		
u						$-\Delta$			
s									$-\Delta$
d								$-\Delta$	

CFL pairing vs. the strange quark mass

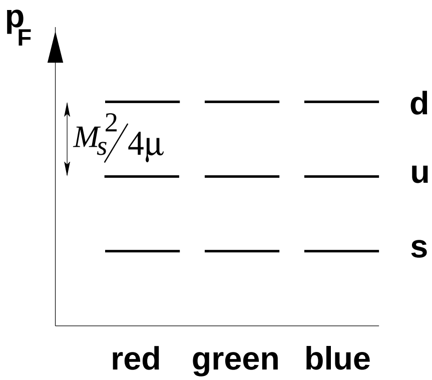
In the real world, Cooper pairing has to compete with stresses that separate the quark Fermi momenta:

1. Strange quark mass is not infinite nor zero, but intermediate. It depends on density, and ranges between about 500 MeV in the vacuum and about 100 MeV at high density.
2. Neutrality requirement. Bulk quark matter must be neutral with respect to all gauge charges: color and electromagnetism.
3. Weak interaction equilibration. In a compact star there is time for weak interactions to proceed: neutrinos escape and flavor is not conserved. $u \rightarrow d\, e^{+}\, \nu_e$, etc.

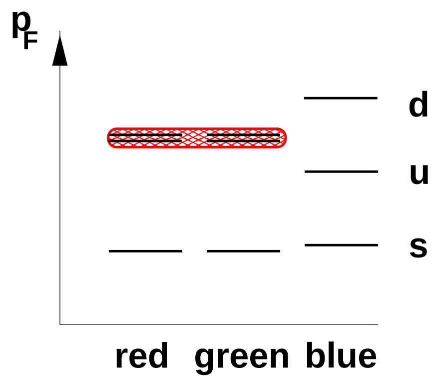
Cooper pairing vs. the strange quark mass

$M_s \sim 100-500$ MeV, $M_{u,d} \sim 5-100$ MeV, $\quad Q_u = +\frac{2}{3} \quad Q_{d,s} = -\frac{1}{3}$

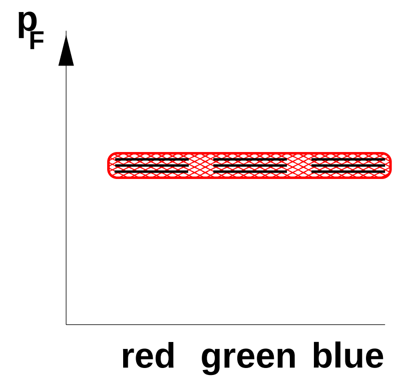
Unpaired



2SC pairing



CFL pairing



CFL: Color-flavor-locked phase, favored at the highest densities.

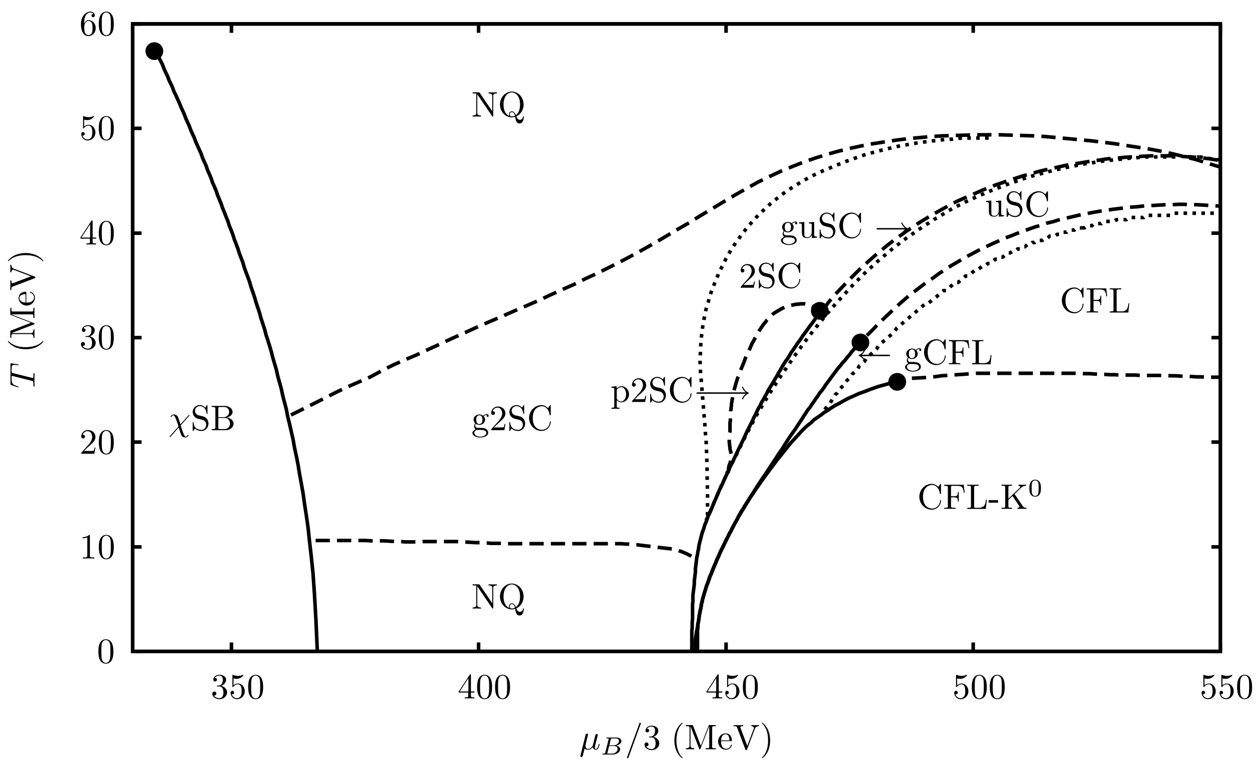
$$\langle q_i^\alpha q_j^\beta \rangle \sim \delta_i^\alpha \delta_j^\beta - \delta_j^\alpha \delta_i^\beta = \epsilon^{\alpha\beta N} \epsilon_{ijN}$$

2SC: Two-flavor pairing phase. May occur at intermediate densities.

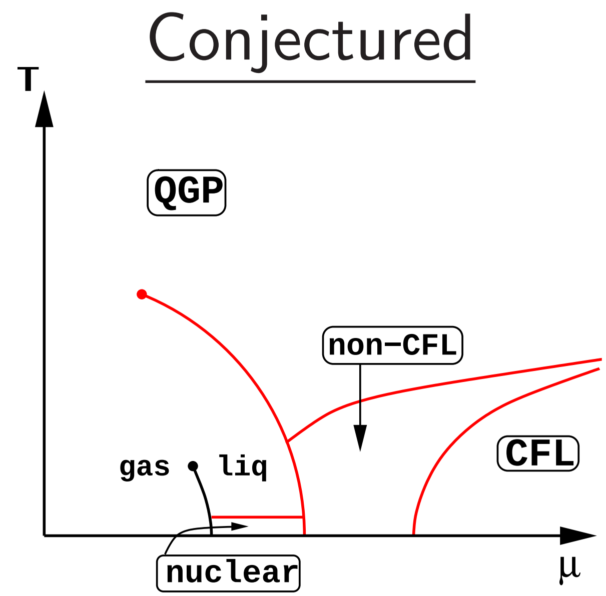
$$\langle q_i^\alpha q_j^\beta \rangle \sim \epsilon^{\alpha\beta 3} \epsilon_{ij3} \sim (rg - gr)(ud - du)$$

or: Exotic non-BCS pairing: FFLO (crystalline phase), p-wave meson condensates, single-flavor pairing (color-spin locking, ∼liq ^{3}He-B).

Phase diagram from one NJL model



Warringa, `hep-ph/0606063`



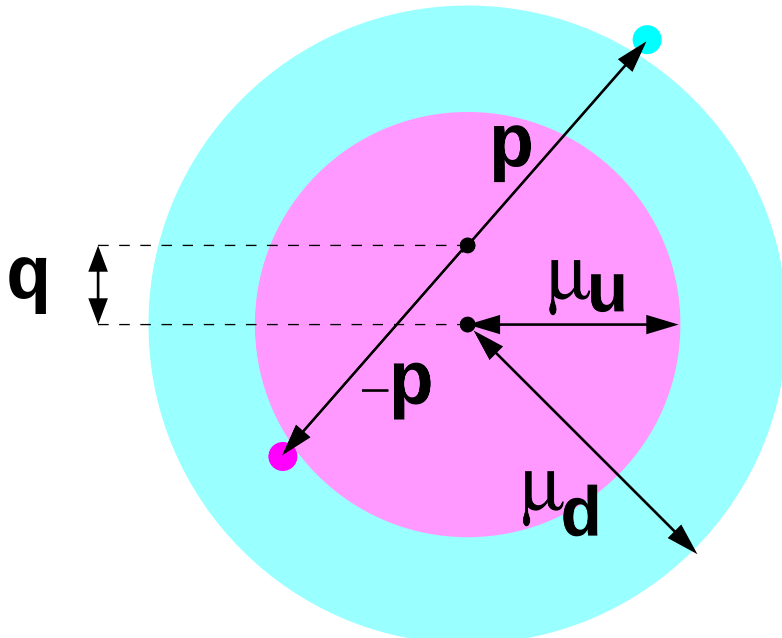
This is only a model. Any resemblance to the real QCD phase diagram is purely coincidental.

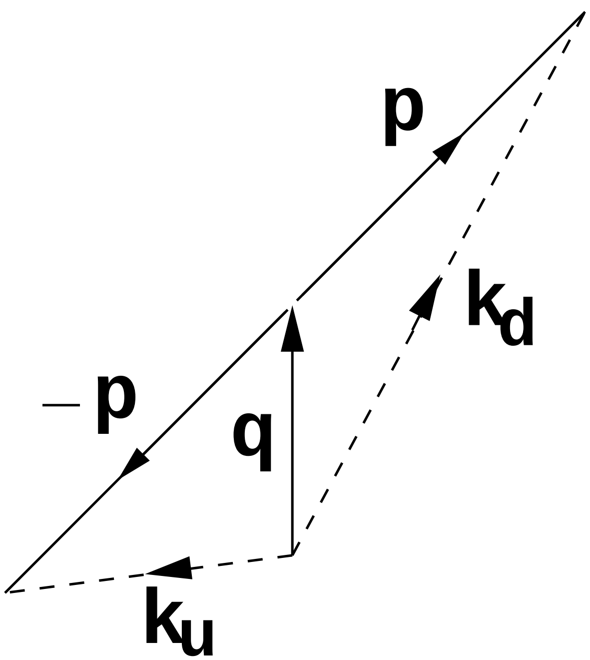
This model only includes isotropic phases.

But gapless isotropic phases (g2SC, gCFL, etc) tend to have negative gradient terms (imaginary Meissner mass) $\Rightarrow$ spontaneous anisotropy develops. For example, the LOFF/FFLO phases. . .

Crystalline (FFLO or LOFF) color superconductivity

When the Fermi momenta are such that one flavor of quark is just barely excluded from pairing with another, it may be favorable to make pairs with a net momentum, so each flavor can be close to its Fermi surface.

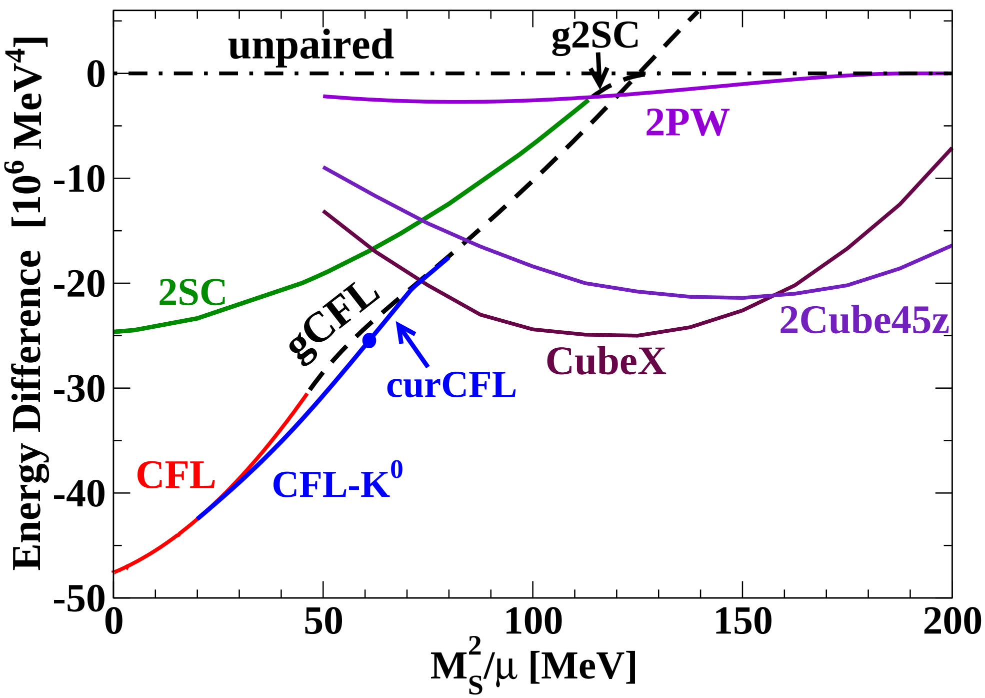




so $\mathbf{p}$ identifies a particular quark pair, and every quark pair in the condensate has the same nonzero total momentum $2\mathbf{q}$ (single plane wave LOFF).

Free energy comparison including anisotropic phases

Assuming $\Delta_{\rm CFL} = 25$ MeV.



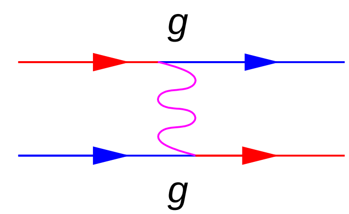
CFL-K^0	K^0 condensate
curCFL	K^0 cond current
2PW	2-plane-wave LOFF
CubeX	LOFF crystal, G-L approx
2Cube45z	LOFF crystal, G-L approx

Alford, Rajagopal, Schäfer, Schmitt, `arXiv:0709.4635`

Curves for CubeX and 2Cube45z use G-L approx at the edge of its area of validity: favored phase at $M_s^2 \sim 4\mu\Delta$ remains uncertain.
Rajagopal and Sharma, `hep-ph/0605316`

Asymptotically high densities

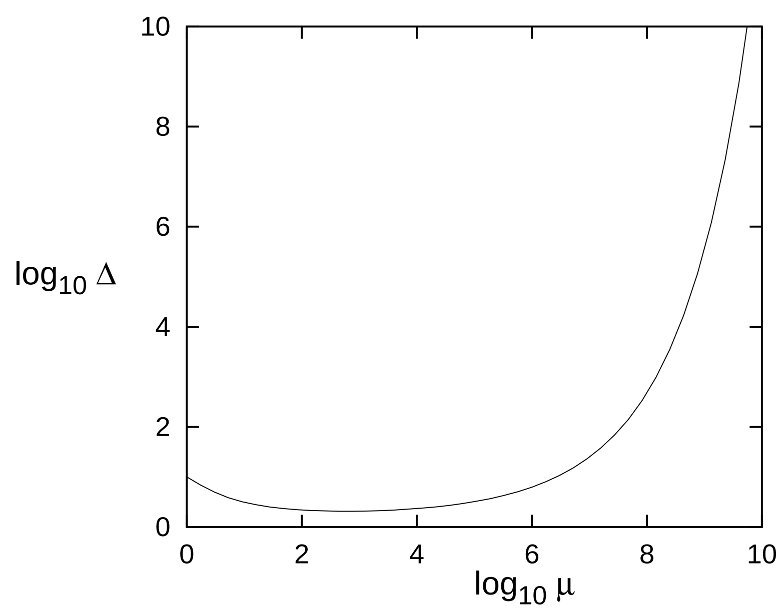
At unrealistically high densities, CFL pairing is favored.



- M_s^2/μ becomes negligible
- All quark modes gapped, all non-Abelian gluons massive, so no infra-red instabilities of CFL ground state
- running coupling $g(\mu) \ll 1$, so weak coupling works

$$\Delta(\mu) \propto \mu g(\mu)^{-5} \exp\left(-\frac{3\pi^2}{\sqrt{2}g(\mu)}\right)$$

Note $\exp(-1/g)$, not $\exp(-1/g^2)$, due to divergent forward scattering from unscreened long-range color-magnetic interaction.



(μ in GeV)

III. Quark matter in compact stars

Where in the universe is color-superconducting quark matter most likely to exist? In compact stars.

A quick history of a compact star.

A star of mass $M \gtrsim 10 M_\odot$ burns Hydrogen by fusion, ending up with an Iron core. Core grows to Chandrasekhar mass, collapses $\Rightarrow$ supernova. Remnant is a compact star:

mass	radius	density	initial temp
$\sim 1.4 M_\odot$	$\mathcal{O}(10 \text{ km})$	$\gtrsim \rho_{\text{nuclear}}$	~ 30 MeV

The star cools by neutrino emission for the first million years.

Signatures of color superconductivity in compact stars

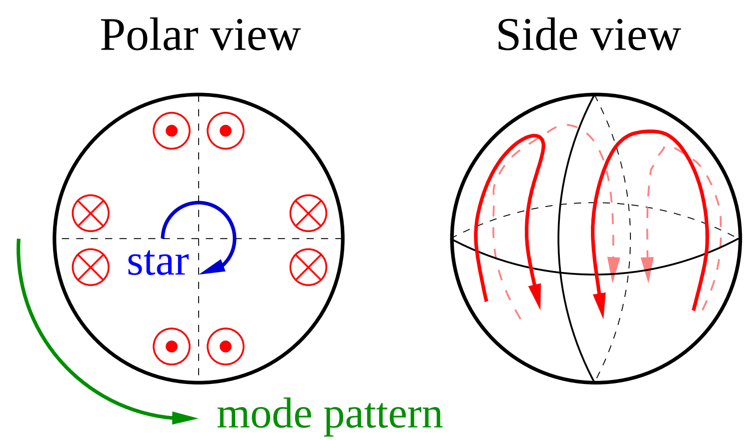
Pairing energy: affects **Equation of state**. Hard to detect.
(Alford, Braby, Paris, Reddy, nucl-th/0411016)

Gaps in quark spectra and Goldstone bosons: affect **Transport properties**:
emissivity, heat capacity, viscosity (shear, bulk),
conductivity (electrical, thermal)...

1. Cooling by neutrino emission, neutrino pulse at birth
   (Page, Prakash, Lattimer, Steiner, hep-ph/0005094; Carter and Reddy, hep-ph/0005228; Reddy, Sadzikowski, Tachibana, nucl-th/0306015; Grigorian, Blaschke, Voskresensky astro-ph/0411619)
2. Glitches and crystalline ("LOFF") pairing
   (Alford, Bowers, Rajagopal, hep-ph/0008208; Mannarelli, Rajagopal, Sharma hep-ph/0702021)
3. Gravitational waves: r-mode instability, shear and bulk viscosity
   (Madsen, astro-ph/9912418; Manuel, Dobado, Llanes-Estrada, hep-ph/0406058; Alford, Schmitt nucl-th/0608019; Alford, Braby, Reddy, Schäfer nucl-th/0701067; Manuel, Llanes-Estrada arXiv:0705.3909)

r-modes: gravitational spin-down of compact stars

An r-mode is a quadrupole flow that emits gravitational radiation. It arises spontaneously when a star spins fast enough, and if the shear and bulk viscosity are low enough.

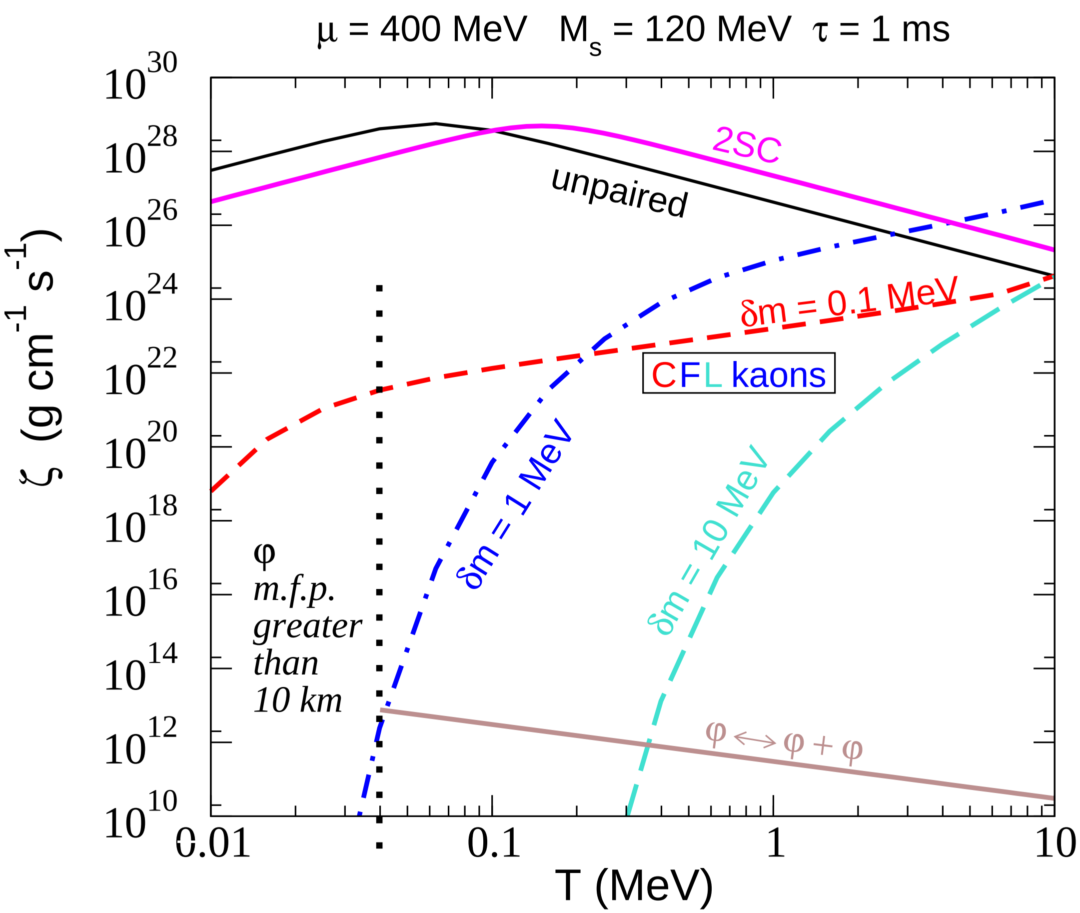


What is bulk viscosity?

(L. *viscum* = mistletoe; It: vischio, Jp: 宿り木, Ger: Mistelzweig, Sp: muérdago, Fr: gui, . . .)
A sticky glue was made from mistletoe berries and coated onto small tree branches to catch birds.

Energy consumed in a compression cycle of period τ. Bulk viscosity arises from re-equilibration processes. If some quantity goes out of equilibrium on compression, and re-equilibrates on a timescale comparable to τ, then pressure gets out of phase with volume and energy is consumed. (Just like V and I in a R-C circuit.)

Bulk viscosity of quark matter phases

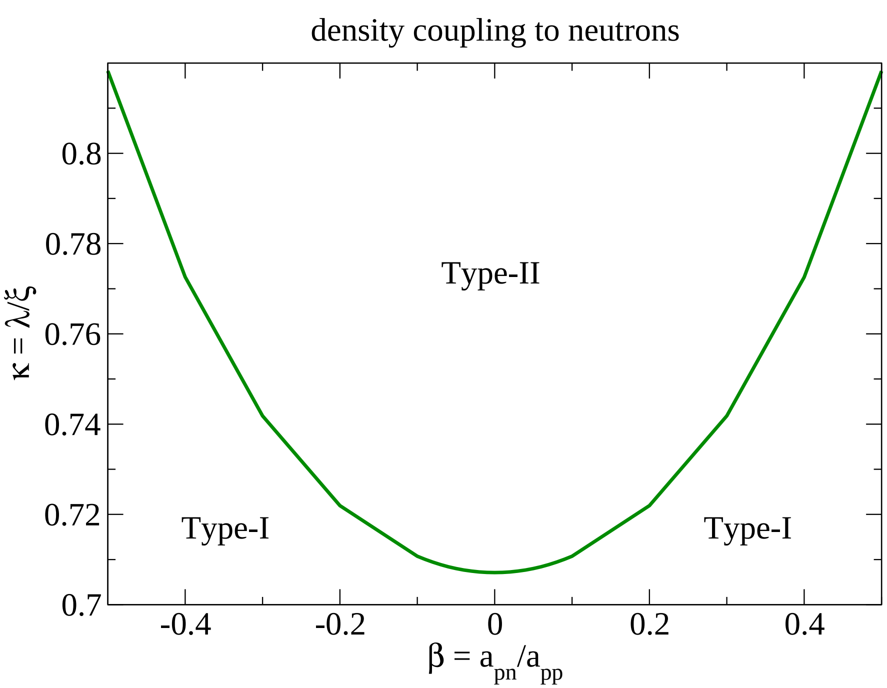


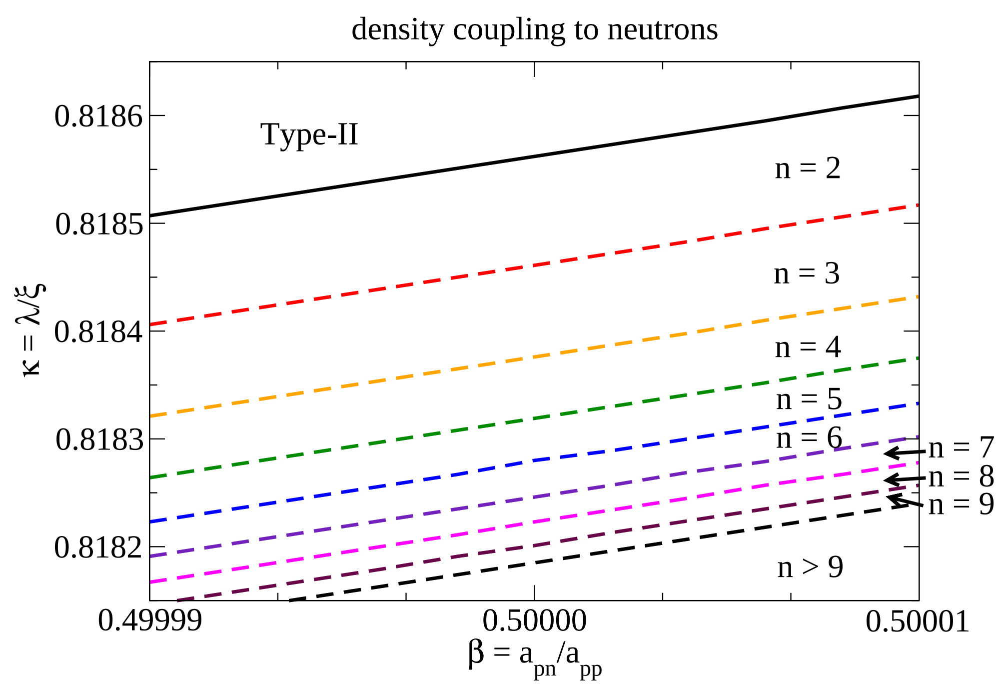
- Unpaired and 2SC have the largest bulk viscosity, because they have unpaired modes at Fermi surface (large phase space).
- K^0 density $\sim \exp(-\delta\mu/T)$ drops rapidly for $T \lesssim \delta\mu/10$.
- $\delta\mu = m_{K^0} - M_s^2/(2\mu)$ could be anything from negative (kaon condensation) to ~ 10 MeV.
- Superfluid modes ("phonons") alone contribute some bulk viscosity.

Alford, Schmitt, `nucl-th/0608019`;
Alford, Braby, Reddy, Schäfer, `nucl-th/0701067`;
Manuel, Llanes-Estrada, `arXiv:0705.3909`

Superconductor coupled to a superfluid

In nuclear matter and sometimes in quark matter, we get coexisting superconductor and superfluid. If there is a coupling β between the condensates, how does that affect the superconductor's flux tubes?





M. Alford and G. Good, unpublished

V. Looking to the future

- Neutron-star physics of color superconducting quark matter:
  - shear and bulk viscosity of 2SC, CFL, other phases...
  - detailed analysis of r-mode profiles in hybrid star
  - heat capacity, conductivity and emissivity (neutrino cooling)
  - structure: nuclear-quark interface (gravitational waves?)
  - crystalline phase (glitches)
  - CFL: vortices but no flux tubes
  - effect of high magnetic fields
- More general questions:
  - magnetic instability of gapless phases
  - better weak-coupling calculations, include vertex corrections
  - go beyond mean-field, include fluctuations
  - solve the sign problem and do lattice QCD at high density.